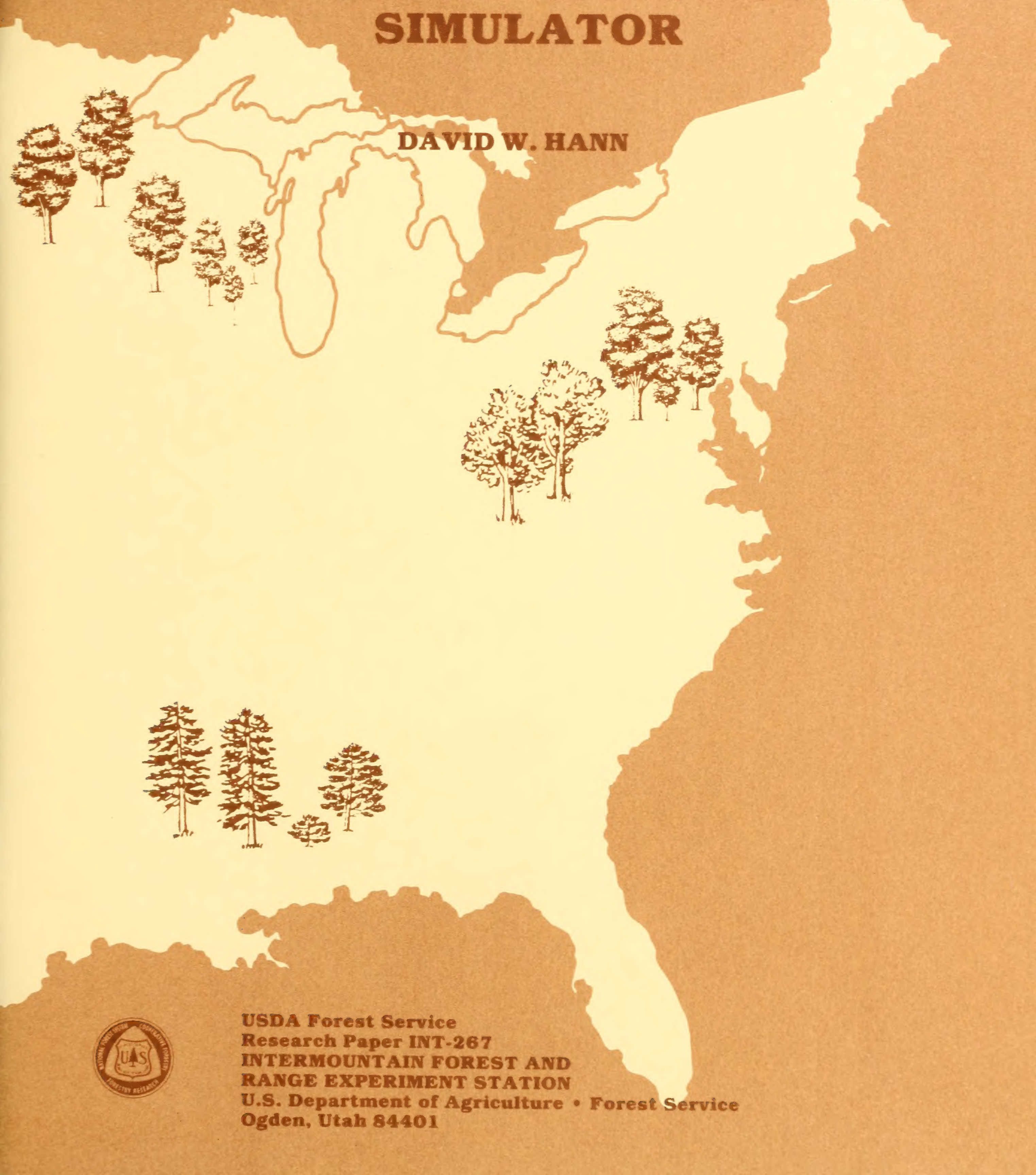
DEVELOPMENT AND EVALUATION OF AN EVEN- AND UNEVEN-AGED PONDEROSA PINE/ARIZONA FESCUE STAND SIMULATOR



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INTERMOUNTAIN FOREST AND
RANGE EXPERIMENT STATION
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RESEARCH SUMMARY

A simulator for predicting even- and uneven-aged stand development was constructed and validated for the ponderosa pine/Arizona fescue habitat type of the Southwest. The structure and dynamics of the simulator make it potentially useful for answering both even- and uneven-aged management questions.

The primary framework for the simulator is two diameter distributions, one for blackjack pine and the other for yellow pine (both are different vigor classes of ponderosa pine). The diameter distributions are expressed as number of trees in each 1-inch diameter class, and the lowest diameter class is 4 inches for uneven-aged stands and 1 inch for even-aged stands. Within this framework, there are four basic components: upgrowth, mortality, conversion, and ingrowth.

Upgrowth is the movement of trees from a given diameter class into larger diameter classes. It was modeled as a function of the distribution of trees within the diameter class and of basal area growth of the diameter class. During modeling, diameter class basal area growth rates of even- and uneven-aged stands were shown to be logically interrelated if the structure of within-stand competition is incorporated in the model.

Diameter class mortality was modeled as the proportion of trees dying to eliminate the problem of negative survival rates. Even- and uneven-aged endemic mortality rates also proved to be logically interrelated if catastrophic mortality was treated in a fashion similar to cutting "mortality."

Conversion is the process in which blackjack pines transform into yellow pines. As with mortality, conversion was modeled as a proportion of the diameter class transforming.

The final component of the simulator is ingrowth, which is the number of trees growing into the 4- or 5-inch diameter classes of blackjack pine. Data restrictions precluded the development of a regeneration model and limited the ingrowth model to uneven-aged stands. For even-aged stands, it is assumed that no (or insignificant) trees exist below the 1-inch diameter class. Simulator applications are further restricted to stands with an average stand diameter of 4 inches or greater.

Two equations were needed to model the ingrowth process. The first predicts the total number of trees ingrowing into the 4- and 5-inch diameter classes. The second equation predicts the proportion of total ingrowth that will through grow into the 5-inch diameter class in the same growth period.

During the validation process, an assumption made by past uneven-aged modelers was tested: that small plots have the same structure and dynamics as the total stand and can therefore be used to develop whole stand simulators. From this analysis, I concluded that developing a simulator, using 2.5-acre plots, and then using it to predict stand averages generally reduces both the precision and accuracy of the stand estimates. In this study, however, the loss appears not too severe for most potential uses of a whole stand simulator.

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STUDY OBJECTIVES

Recent publications (USDA Forest Service 1975, 1976; Alexander and Edminster 1977a, 1977b; Hann and Bare 1979) suggest a renewal of interest in uneven-aged forest management. Hann and Bare (1979) reviewed major questions facing a forest manager interested in practicing uneven-aged management and summarized the analytical tools currently available to answer these questions. They demonstrated the important role stand development simulators play in making decisions using some of the more recent analytical tools. The construction of appropriate stand development simulators is therefore a prerequisite to the application of these analytical tools.

A number of simulator types have been developed in recent years (Munro 1974). But technological limitations of the available uneven-aged analytical tools require using whole-stand simulators that characterize the stand by at least the number of trees in various diameter classes (Adams 1974; Adams and Ek 1974, 1976; Hann 1978). With this constraint, the objectives of this study became:

1. To examine the dynamics of uneven-aged stand development in the ponderosa pine/ Arizona fescue habitat type by developing a whole-stand simulator that would have the potential for aiding forest managers in answering questions concerning uneven-aged management. Ponderosa pine was chosen because it is one of the few species in the West for which permanent plot data exist for both even- and uneven-aged stands.

2. To examine whether even- and uneven-aged stand development in ponderosa pine/Arizona fescue could logically be interrelated and, if so, to develop the whole-stand simulator to be applicable to both.

3. To validate the whole-stand simulator by analyzing its capability to predict both average plot and average stand development.

PREVIOUS UNEVEN-AGED, WHOLE-STAND SIMULATORS

Two approaches to the development of uneven-aged, whole-stand simulators have been reported. The first approach uses differential equations and the second uses difference equations to characterize stand dynamics. The differential equation approach is based on using instantaneous rate equations to model stand dynamics. Two alternate techniques exist for parameter estimation of the instantaneous rate equations. In the first technique, modelers approximate the instantaneous rates through use of periodic rates that have been converted to an annual basis, and then estimate the parameters through regression analysis (Moser and Hall 1969; Moser 1972, 1974). If the length of the growth period is short and if the instantaneous rates do not change drastically within the period, then this technique can provide satisfactory estimates of the instantaneous rate equations. The second technique uses the multipoint boundary value method to develop parameter estimates of the instantaneous rate equations (Leary 1970). This process is a common numerical analysis method but requires data from at least three different points in the development of the stand. While Leary's (1970) use of this method was in even-aged stands, it would also be appropriate in uneven-aged stands.

The second approach recognizes that most forest rate data are periodic in nature and, therefore, an appropriate way to model them is by using difference equations. An example of an uneven-aged, whole-stand simulator using this approach is Ek's (1974) mixed species, northern hardwood simulator that characterizes the stand by number of trees in 2-inch diameter classes. In this simulator, the number of trees in each diameter class at the end of a growth period is a linear combination of ingrowth, upgrowth, and mortality. Ingrowth is the number of trees growing into the smallest diameter class in a growth period, and Ek modeled it as a nonlinear function of total stand basal area and total number of trees. Upgrowth is the number of trees growing out of a specified diameter class in a growth period, and it was modeled as a nonlinear function of number of trees and basal area in the diameter class, total number of trees and basal area in the stand, and site index. Mortality is the number of trees in a specified diameter class that die in a growth period, and it was modeled as a nonlinear function of number of trees and basal area in the diameter class, and total number of trees and basal area in the stand. Equations were also developed for predicting rough cordwood volume and board-foot volume. Cuttings were introduced by reducing the number of trees in the appropriate diameter class at the end of the growth period.

As a demonstration of the application of the uneven-aged analytical decisionmaking tools they had developed, Adams and Ek (1974) used a modified version of Ek's (1974) model.

The difference equation approach was used in this study for two reasons. First, the development of difference equations appeared to be the most logical use of the periodic data available. Because the data source and the equation type are both periodic, it is not necessary to make numerical approximations in order to develop and/or apply the equations. Most models using differential equations do require numerical approximation methods to estimate parameters and/or to solve them.

Second, the use of difference equations avoids potential problems of inappropriately applying the equations to growth periods different from that used in equation development. Because of the continuity of differential equations, it is possible to predict growth for any period. The differential equations are usually, however, approximations of the true continuous growth process (not only can the parameter estimates be approximations, but the model forms themselves are often approximations). As such, they should not be applied to growth periods much different from those used in equation development or else erroneous results can occur. On the other hand, in normal use difference equations can only be used to predict growth for periods equal to those used to develop the equations. Multiple periodic growth (or fractional periodic growth, if the user wishes to chance extending the equations in that fashion) is derived from these periodic growth rates.

STUDY DATA BASE

Plot Characteristics

The data used in this study were from the Fort Valley Experimental Forest, located a few miles northwest of Flagstaff, Arizona. The uneven-aged data consist of long-term remeasurements on four 80-acre plots. Each plot has been divided into 32 or 34 subplots ranging in size from 0.5 to 3.1 acres, but most of these subplots average near 2.5 acres in size. The measurement of the plots started either in 1920 or 1925, and remeasurements have occurred at 5-year intervals until the 1960's when the interval was lengthened to 10 years. The two attributes recorded on every tree above the lower diameter limit were diameter at breast height (d.b.h.) and tree condition (a classification as to presence of a damaging agent if the tree is alive, or to the cause of mortality if the tree has died within the previous growth period). Age class and age-vigor class of each tree were measured every fourth growth period. Age class categorizes a tree as being blackjack or yellow pine, and age-vigor uses age and size information, in combination with the previous 20-year average diameter growth rate, to help identify the tree's growth potential. The lower diameter limit was first set at 3.6 inches (the lower bound for the 4-inch diameter class). It was then changed to 7.6 inches in 1940, and recently to 6.0 inches. A brief history and description of each uneven-aged plot is in table 1.

Table 1.--*History and description of uneven-aged study plots*

Plot No.	Description
61	Represents virgin ponderosa pine. The south half of the plot was first measured in 1920 and the other half in 1925. Average site index of this plot has been estimated to be approximately 67 by Minor's (1964) site curve.
62	First measured in 1920. It was salvage cut in 1940 and again in 1953. Average site index is 73 (Minor 1964).
71	Had a modified selection cut in 1924, followed by its first measurement in 1925. A timber stand improvement (T.S.I.) cut occurred in 1935, an improvement selection cut in 1946, a precommercial thinning in 1967, and a group selection cut in 1968. The basal area stocking level now established as a goal for this plot is 96 square feet. Average site index is 73 (Minor 1964).
72	Treatment has been identical to that of plot 71. The basal area stocking level now set for this plot, however, is 70 square feet. Average site index is 70 (Minor 1964).

The even-aged data came from the Taylor Woods level of growing stock study, located very close to the uneven-aged plots (Schubert 1971), and consisting of 18 plots ranging in size from 0.75 to 1.25 acres. These plots represent three replications of six growing stock levels (30, 60, 80, 100, 120, and 150 square feet of basal area). The plots were measured in 1962, 1967, and 1972. They were thinned in 1962 and 1972 (the 1962 thinning was before measurement). All trees on the plots were measured. Average site index for the study area is 88 (Minor 1964).

The attributes measured on each tree include d.b.h., tree condition, crown class (whether a tree was a dominant, codominant, and so forth), crown quality (the condition of the crown), and percent of the bole in live crown. On a subsample of the trees, the following additional items were measured: total tree height, crown length, and crown width.

Data Problems

While this data source is unusual for the West because it provides both even-aged data and long-term uneven-aged growth data, it also posed numerous modeling problems. A brief discussion follows on these problems and the actions taken to minimize them. See appendix A for a more complete discussion.

1. All of the data used in this study came from a narrow geographic and site range. Therefore, it is recommended that the simulator developed in this study be applied only to stands on the ponderosa pine/Arizona fescue (*Pinus ponderosa/Festuca arizonica*) habitat type of northern Arizona.

2. Because of changes in the lower limit of diameter measurements, only data measured up to 1940 were used to estimate component model parameters.

3. An intrusion of a major paved highway through all of the uneven-aged plots necessitated the removal of some subplots.

4. As mentioned earlier, a wide range of subplot sizes exists on the uneven-aged plots. To avoid a potential problem of confounding subplot stand differences with subplot size differences, all uneven-aged subplots below 2 acres were eliminated from the data base.

5. Because of a lack of height-growth data on the uneven-aged plots, development of a height-growth model was not possible. Instead, additional data needed for the development of height-diameter equations were obtained from the School of Forestry at Northern Arizona University and from the Rocky Mountain Forest and Range Experiment Station at Flagstaff.

Plot and Subplot Allocation

With the elimination of the small subplots and those seriously affected by the highway, 110 uneven-aged subplots remained. Of these, 24 were reserved for validation (objective number 3). To pick these, each of the four uneven-aged plots had their subplots divided into low, medium, and high basal area stocking classes (one-third of the subplots in each), and two subplots were randomly selected from each class. This helps to insure that the validation data represent the full range of stocking conditions.

Also reserved for validation were six even-aged plots. These were also randomly chosen by selecting one plot from each of the six basal area stocking classes. The result was 86 uneven-aged and 12 even-aged plots for use in developing the simulator.

SIMULATOR DEVELOPMENT

The development of a whole-stand simulator that can express number of trees by diameter class is the main objective of this study. A 1-inch diameter class size was chosen because it offered a good compromise between model complexity on one hand and sensitivity to stand structure and to managerial and silvicultural information needs on the other. The decision to express the stand as number of trees instead of basal area in each diameter class was made because many of the simulator's stand dynamic components are best expressed in terms of number of trees.

The dynamics of stand development is composed of three basic elements: growth, mortality, and recruitment.

Growth

The process of growth takes place on all portions of the tree: roots, crown, and stem. The growth of most interest to the manager, however, is stem or bole growth, which is the height and diameter growth. With the lack of height-growth information in the data set, a height-growth model could not be developed. Instead, height equations were developed as a substitute to aid in the prediction of product potential. Lack of a height-growth model did not affect the processes used to predict change in number of trees by diameter class. Such a change is the difference between the number of trees growing into the diameter class and the number growing out of it. The number of trees growing from one diameter class to another, sometimes called upgrowth, is a function of three components: diameter growth, the distribution of number of trees within the diameter class, and diameter class size (Wahlenberg 1941).

At least two approaches exist to modeling this process. In one approach, Ek (1974) chose not to treat these components separately, but rather, he predicted the number of trees growing into the next larger diameter class directly. Another approach is to consider each component of the upgrowth model individually. This approach was used in the study because it allowed a closer examination of the validity of the individual component assumptions incorporated into the upgrowth model.

DISTRIBUTION WITHIN DIAMETER CLASS AND DIAMETER CLASS SIZE

The distribution of the trees within a diameter class can influence the number of trees advancing out of the class, given an average class diameter growth rate (Wahlenberg 1941; Meyer 1942). The easiest distribution to use, and the one assumed by Moser (1974), is the uniform distribution. If this distribution is valid, then the following method can be used for computing upgrowth (Wahlenberg 1941; Meyer 1942). The quotient found by dividing diameter growth by diameter class size is computed. The integer portion of this quotient is then the number of diameter classes in which all trees will advance, and the fractional portion of the quotient is the proportion of the trees that will advance one diameter class further. (For example, if the quotient was 2.29, then all trees will advance two diameter classes, and 29 percent of them will advance three diameter classes.)

Because it is the easiest to apply, I first tested the appropriateness of the uniform distribution through use of the chi-square "goodness-of-fit" test (see appendix B for details). A total of 714 diameter classes were tested at the 99 percent level of significance, and of these, only seven were found to be significantly different from the uniform distribution. These findings were further substantiated by visually checking the distributions of many of these classes. Therefore, the assumption of within-class uniformity could not be rejected in the even- and uneven-aged stands of the Fort Valley Experimental Forest.

DIAMETER GROWTH

The final component necessary for the calculation of upgrowth is an estimate of the diameter growth rate for each class. The following summarizes procedures used to develop the appropriate diameter growth models. A more complete discussion is in appendix C.

Initial Review and Modeling Decisions

A review of the literature for southwestern ponderosa pine helped identify factors historically recognized as related to diameter growth. The significant factors identified included: site index, rainfall, total stand density, diameter class size, tree vigor as indicated by bark color (in other words, "yellow" pine and "blackjack" pine), and the structure of the competition as indicated by the position of the diameter class in the stand.

Given the general factors that can affect diameter growth, it was then necessary to choose: the general structure of the model, the form of the dependent variable, the method for handling the error structure of the model during prediction, and the appropriate transformations of the independent variables. After careful consideration, I decided to predict the natural log of diameter class basal area growth and then convert that value to a diameter growth rate. Three basic reasons for this choice are:

1. A nonlinear, multiplicative error model form probably best represented the interaction of the independent variables with themselves and their effect upon diameter growth.

2. Basal area growth is often nearly linear over short time periods, which simplifies the extrapolation of growth rates to growth periods different from that originally used in equation development.

3. The residuals of log of basal area growth more often approach normality and homogeneous variance than other dependent variables, which is advantageous in model development and significance testing.

Several ways of using information about the error structure of the model are available when using the finished model for predictive purposes. In deterministic modeling, the random error element of the model is simply ignored for predictive purposes. If the model is properly constructed, predictions from a deterministic model represent the "expected" value of the output. In stochastic modeling, the random error element is considered in prediction. To obtain an expected value of the output from a stochastic model, however, requires a "Monte Carlo analysis," which can be time consuming and expensive.

Stage (1973) described an alternative method, which could be termed a "Monte Carlo swindle," of introducing a stochastic element into his individual tree log of basal area growth equations while maintaining the relative simplicity of a deterministic model. Because it was expected that individual tree growth within a diameter class would be quite variable, elements from Stage's (1973) "Monte Carlo swindle" were adapted and used in this study.

The resulting process consists of dividing the number of trees in each diameter class into thirds (one each to represent the fast, slow, and "moderate" growers), predicting the expected residual for each third, and adding or subtracting this to the predicted average log of basal area growth value for the diameter class. In this fashion, estimates of basal area growth are obtained for each third of the diameter class. Upgrowth is then calculated separately for each third, which allows the trees in a diameter class to move into a wider range of larger diameter classes. More realistic predictions should result.

The final decision before parameter estimation could begin was the choice of appropriate transformations necessary to create those independent variables most likely to be highly correlated to the dependent variable. Information used in selecting the independent variables included the silvicultural literature on diameter growth factors for southwestern ponderosa pine and prior general "growth and yield" knowledge reported in the mensurational literature. From this work, 43 potentially useful independent variables were identified and examined in the next phase of the diameter growth modeling process.

Development of Equations

Ordinary, least squares regression techniques were used to develop separate equations for both blackjack and yellow pine. I initially separated these two vigor classes because evidence from prior studies suggest significant growth differences between blackjack and yellow pine. The analysis process to determine the form and parameters of the finished models required six phases:

Phase 1. An all-combinations screening run was made on each individual plot data set to eliminate those independent variables not highly correlated to the dependent variable.

Phase 2. The separate plot data were then combined into three sets: virgin, uneven-aged data; managed, uneven-aged data; and managed, even- and uneven-aged data. A second set of all combination screening runs was then made for each data set and for yellow and blackjack pine. From these runs, a common set of independent variables exhibiting behavior both reasonable and consistent with mensurational and silvicultural expectations was selected for both yellow and blackjack pine. Because the basic model forms and the size of the parameter estimates differed so greatly between yellow and blackjack pine, I decided that at this phase of the analysis any attempt to combine the two vigor classes through use of dummy variables would be futile. Therefore, separate equations were maintained throughout the rest of the analysis.

Phase 3. The combined data sets were further collapsed into two sets: uneven-aged and even- and uneven-aged. The reason for maintaining separate sets was to allow comparison of the effect upon predictive capability of adding the even-aged data to the uneven-aged. To do this merging, the differences between the model coefficients for the managed and the virgin data sets were handled as functions of time-since-last-cutting. Sigmoidal transformations of time-since-last-cutting were used because the effect of cutting was expected to asymptotically approach the virgin growth rate as time-since-last-cutting increased. Combining these new variables with those already selected, another set of screening runs was made from which the most promising equations were selected and the regression coefficients determined. Examination of these coefficients showed the signs on several of the transformations were not reasonable (including the site index transformations), and that a high degree of multicollinearity existed.

Phase 4. Ridge regression was then used as a means of reducing the problems of multicollinearity. I hoped the minimization of multicollinearity might reverse the problem with sign on the site index transform, but the resulting analysis did not help. Because of the desire to have a model applicable over the range site indices naturally found in the habitat type, I decided to force a reasonable value of the site transform upon the model. This was done by defining a new dependent variable--the natural log of the quotient basal area growth divided by site index. By using this dependent variable, I assumed, therefore, that basal area growth would increase in a direct proportion to an increase in site index.

Phase 5. A final set of screening runs was then made with the new dependent variable using the same methods described in the third phase of the analysis. From these screening runs, I selected promising models and made additional runs to determine their regression coefficients.

Phase 6. Ridge regression was then used to selectively remove several of the time-since-last-cutting independent variables in order to minimize multicollinearity. Parameters to the final models were then determined by ordinary, least squares regression.

Log of Basal Area Growth Equations

The finished log of basal area growth equations is in table 2. The two blackjack pine equations differ because of the presence or absence of the even-aged data when they were developed. The number of observations, the relative mean square residual (RMSQR),[1] and the coefficient of determination (R^2) of each equation are found in table 3.

Table 2.--*Log of basal area growth equations*

Blackjack pine using all data:

$$\begin{aligned} ln(\text{Basal Area Growth}) &= -8.51836897 + 1.16754330(ln(D)) - 4.00970143E\text{-}02(D) \\ &\quad - 3.84298771E\text{-}03(LBA_2) - 7.15483662E\text{-}03(MBA_2) - 1.58234269E\text{-}02(UBA_2) \\ &\quad - 3.26097273E\text{-}01(A_1 ln(D)) + 8.80676713E\text{-}01(A_3) + 1.0(ln(S)) \end{aligned}$$

Blackjack pine using uneven-aged data:

$$\begin{aligned} ln(\text{Basal Area Growth}) &= -8.45357088 + 1.18165715(ln(D)) - 4.77068027E\text{-}02(D) \\ &\quad - 1.25420926E\text{-}03(LBA_2) - 7.69300113E\text{-}03(MBA_2) - 1.74056839E\text{-}02(UBA_2) \\ &\quad - 3.13247572E\text{-}01(A_1 ln(D)) + 9.10604169E\text{-}01(A_3) + 1.0(ln(S)) \end{aligned}$$

Yellow pine:

$$\begin{aligned} ln(\text{Basal Area Growth}) &= -15.2464932 + 4.27656656(ln(D)) - 1.83161626E\text{-}01(D) \\ &\quad - 7.27361567E\text{-}05(LBA_2)^2 - 9.11165626E\text{-}04(MBA_2)^2 - 2.41462106E\text{-}04(UBA_2)^2 \\ &\quad - 1.05776062(A_1) + 1.0(ln(S)) \end{aligned}$$

where

D = diameter class size

S = Minor's (1964) site index

MBA_2 = basal area in the given diameter class plus the two adjoining larger and smaller diameter classes

LBA_2 = total basal area below the smallest diameter class in MBA_2

UBA_2 = total basal area above the largest diameter class in MBA_2

$A_1 = -0.244178 + 1.244178*EXP(-(1.176471 - 0.019607*TIME)^3)$

$A_3 = -0.000203 + 1.000203*EXP(-(1.428571 - 0.023809*TIME)^6)$

TIME = number of 5-year growth periods since last cutting

[1]Relative mean square residual is the mean square residual for the model divided by the corrected variance of the dependent variable. See appendix C for more detail.

Table 3.--*Statistics for log of basal area growth equations*

Equation type	Number of observations	RMSQR	R^2	Modified RMSQR	Modified R^2
Blackjack pine with even-aged data	33,155	0.8001	0.2000	0.5155	0.4839
Blackjack pine without even-aged data	27,268	.8223	.1779	.5425	.4576
Yellow pine	4,558	.9334	.0678	.8847	.1165

Because of the presence of within diameter class variation, the statistics quantifying the fit of the equations are unduly pessimistic. The equations explain between class variation and, therefore, the statistics were modified by eliminating the within diameter class variation component to better reflect the true fit of the equations. These "modified" statistics are also found in table 3.

The results show that the blackjack pine equations developed with the even- and uneven-aged data explained over 48 percent of the between diameter class variation, the other blackjack pine equation explained over 45 percent, and the yellow pine equation explained only about 12 percent. For the blackjack pine, however, the equations do better at predicting managed growth than virgin growth. I obtained the statistics in table 4 by computing the residuals of each indicated data set around the final models. These statistics show that the blackjack pine equation developed with the even- and uneven-aged data, explained only about 19 percent of the virgin uneven-aged variation, but it explained over 63 percent and almost 78 percent of the variation for the managed uneven- and even-aged data sets, respectively. The other blackjack pine equations explained a little over 19 percent of the virgin and almost 64 percent of the managed uneven-aged variation. Because the managed stand is of primary concern, these results are important.

Table 4.--*Modified coefficients of determination for various data sets by equation type*

Equation type	Modified R^2		
	Virgin uneven-aged data	Managed uneven-aged data	Managed even-aged data
Blackjack pine with even-aged data	0.1941	0.6334	0.7780
Blackjack pine without even-aged data	.1922	.6364	--
Yellow pine	.0440	.0873	--

The poor results for yellow pine were not totally unexpected. These trees are overmature and frequently afflicted by various damaging agents that influence growth rates. In addition, the relatively small number of yellow pine in the stand often meant that between diameter class differences are actually between individual tree differences. Two mitigating factors, however, argued for retaining the equation. First, the model behaves as expected and does predict somewhat better than the mean. Second, the accurate prediction of yellow pine is not very important in managed stands because, under management, the presence of yellow pine usually would not be a favored condition.

Check for Normality and Homogeneous Variance

A check of the final equations revealed that the residuals were highly skewed and leptokurtic, which was the same result as first found for the individual plot equations (appendix C). Homogeneity of variance was checked by dividing the range of predicted log of basal area growth into intervals, computing a variance for each interval, and then comparing the variances across intervals for trends. Bartlett's chi-square test for homogeneity of variances (Snedecor and Cochran 1967) was not used because the test is highly sensitive to nonnormality. The visual checks for homogeneity indicated no consistent trends, and so weighting was judged unnecessary.

Log Bias

The usage of the log model to predict basal area growth introduces a problem because the value of interest for predictive purposes is basal area growth and not the log of basal area growth. The problem arises because $\mathrm{EXP}[\widehat{ln(\mathrm{Y})}]$ is not a "mean-unbiased" estimator of Y; i.e.:

$$\mathrm{EXP}[\mathrm{E}[\widehat{ln(\mathrm{Y})}]] \neq \mathrm{E}[\mathrm{Y}]$$

Rather, $\mathrm{EXP}[\widehat{ln(\mathrm{Y})}]$ is a "median-unbiased" estimator of Y (Bradu and Mundlak 1970).

Because the residuals are not normally distributed, the log bias correction factors proposed by Bradu and Mundlak (1970), Oldham (1965), and Baskerville (1972) could not be used. Therefore, I proposed an alternative log bias correction factor that was added to the intercept term of the log of basal area growth model and was computed as the difference between the log of mean basal area growth minus the mean log of basal area growth (see appendix D).

Models of Mean Residuals

Because the residuals are not normally distributed, the mean deviation of the fast, moderate, and slow growers had to be computed empirically. I hypothesized that the range in residuals would increase as the time-since-last-cutting increased because cutting would, if properly applied, homogenize the stand by eliminating rough, cull, suppressed, and damaged trees. The residuals were, therefore, divided into time-since-last-cutting classes, and for each class the means of the upper, middle, and lower one-third residuals were computed. As expected, the means had a tendency to increase as time-since-last-cutting increased. Therefore, the time-since-last-cutting transforms (A_1, A_2, and A_3) were used to develop models that predict mean residuals for the upper and middle thirds as a function of time-since-last-cutting. The lower one-third mean model was then expressed as the negative of the sum of the other two models. This assumed that, for every value of time-since-last-cutting, the means of the three will sum to zero. The final residual models are in table 5.

Table 5.--*Models for the mean of upper, middle, and lower one-third of residuals about the indicated log of basal area growth model*

Blackjack pine with all data:

$$\text{Upper Mean} = +0.56557130 + 0.12995917A_2$$
$$\text{Middle Mean} = +0.09120012 + 0.12946423A_2$$
$$\text{Lower Mean} = -0.65677142 - 0.25924340A_2$$

Blackjack pine with uneven-aged data:

$$\text{Upper Mean} = +0.61870031 + 0.08118138A_3$$
$$\text{Middle Mean} = +0.13230685 + 0.08389430A_3$$
$$\text{Lower Mean} = -0.75100716 - 0.16507568A_3$$

Yellow pine:

$$\text{Upper Mean} = +0.70338067 + 0.7280557A_3$$
$$\text{Middle Mean} = +0.24266226 + 0.59168455A_3$$
$$\text{Lower Mean} = -0.94604293 - 1.31974025A_3$$

where

$$A_1 = -0.244178 + 1.244178*\text{EXP}(-(1.176471 - 0.019607*\text{TIME})^3)$$
$$A_2 = -0.095336 + 1.095336*\text{EXP}(-(1.250000 - 0.020833*\text{TIME})^4)$$
$$A_3 = -0.000203 + 1.000203*\text{EXP}(-(1.428571 - 0.023809*\text{TIME})^6)$$

Prediction of Basal Area Growth and Diameter Growth

Basal area growth of the one-third fastest (BAG_1), moderate (BAG_2), and slowest (BAG_3) growing trees is predicted by:

$$BAG_i = \text{EXP}[R_i + K + ln(BAG)],\ i = 1, 2, \text{ or } 3$$

where:

BAG_i = predicted basal area growth for the *i*th third of the residuals

R_i = predicted mean log of basal area growth residual for the *i*th third of the residuals

K = log bias correction factor

$ln(BAG)$ = final, average log of basal growth model

Diameter growth of each third can then be computed by the relationship

$$DG_i = \sqrt{\frac{576}{\pi}(DBA + BAG_i)} - \sqrt{\frac{576}{\pi}DBA}$$

where:

DG_i = predicted diameter growth for the *i*th third of the residuals

D = diameter class size

DBA = average basal area of the diameter class

$$= \frac{\pi}{576}(D^2 + 0.1D + 0.085)$$

The equation for the average basal area of a diameter class assumes that the trees in a diameter class are uniformly distributed over the 1/10-inch subclasses of the class. As previously shown, this assumption could not be disproven and therefore would appear correct.

An Initial Evaluation of the Log of Basal Area Growth Equations

An unanswered question is whether basal area growth for even- and uneven-aged stands is logically interrelated. As mentioned previously, the problems with the data used in this study and the lack of normality make a statistical answer to this question intractable. I thought, however, that an initial comparison between the two blackjack pine equations could be made in such a fashion as to shed light on the performance of each model. A second comparison and a selection of a final blackjack pine model were reserved until validation.

To make this initial comparison, the data used to develop the equations were divided into four sets: virgin uneven-aged data, managed uneven-aged data, managed even- and uneven-aged data, and managed even-aged data. The residuals about each final log of basal area growth equation were then computed separately for each of the data sets originally used to develop the particular equation. From this information, a mean squared error (MSE), average residual, and percent average residual (expressed as percent of the mean log of basal area growth) were computed for each data set. The results are in tables 6 through 9.

Table 6.--*Comparison of actual log of basal area growth data from virgin, uneven-aged plots to log of basal area growth predicted by various equations developed using data from those plots indicated*

Vigor class	Plots used in equation development	Number of observations	Mean squared error	Average residual	Percent average residual
Blackjack pine	All	8,197	1.572575	+0.000013	-0.000462
Blackjack pine	Uneven-aged	8,197	1.573900	+ .000028	- .000994
Blackjack pine	Virgin, uneven-aged	8,197	1.534644	+0	0
Yellow pine	Uneven-aged	3,762	6.310643	+ .000261	- .000076
Yellow pine	Virgin, uneven-aged	3,762	6.273243	+0	0

Table 7.--*Comparison of actual log of basal area growth data from managed, uneven-aged plots to log of basal area growth predicted by various equations developed using data from those plots indicated*

Vigor class	Plots used in equation development	Number of observations	Mean squared error	Average residual	Percent average residual
Blackjack pine	All	19,069	0.940228	-0.007303	+0.257221
Blackjack pine	Uneven-aged	19,069	.938867	- .000012	+ .000423
Blackjack pine	Managed, uneven-aged	19,069	.935147	0	0
Yellow pine	Uneven-aged	794	1.840926	- .001237	+ .058064
Yellow pine	Managed, uneven-aged	794	1.800258	0	0

Table 8.--*Comparison of actual log of basal area growth data from managed, even-aged plots to log of basal area growth predicted by various equations developed using data from those plots indicated*

Vigor class	Plots used in equation development	Number of observations	Mean squared error	Average residual	Percent average residual
Blackjack pine	All	5,886	0.224909	+0.023643	-0.814989
Blackjack pine	Managed, even-aged	5,886	.187551	0	0

Table 9.--*Comparison of actual log of basal area growth data from managed, even- and uneven-aged plots to log of basal area growth predicted by various equations developed using data from those plots indicated*

Vigor class	Plots used in equation development	Number of observations	Mean squared error	Average residual	Percent average residual
Blackjack pine	All	24,956	0.771353	-0.000004	+0.000140
Blackjack pine	All managed	24,956	.767860	0	0

Also in those tables are the MSE's, average residuals, and percent average residuals for selected equations developed in phases 2 and 3 of the analysis using just the data from the given set. The equations chosen for comparison were those minimizing RMSQR for each data set. In choosing these equations, no concern was given to whether they were reasonable in behavior, and most of them were not. These equations do, however, represent the best that could be done for the given data set and, therefore, provide a benchmark for how good the final equations are at predicting growth.

For blackjack pine, the results for the virgin uneven-aged data (table 6) indicate that the two equations are quite similar at predicting growth. For the managed, uneven-aged data (table 7), the equation developed with the even-aged data shows a larger (but still small) average residual, but the MSE's are almost the same. This positive percent residual is offset by a negative percent residual for the even-aged data (table 8). When the managed even- and uneven-aged data are put together (table 9), the average residual becomes insignificant. The conclusion, therefore, is that the two blackjack equations give about the same results as far as MSE's and average residuals are concerned.

For yellow pine, the results indicate not much predictive capability was lost when the two data sets (virgin and managed uneven-aged data sets) were combined.

Mortality

Mortality is often separated into a catastrophic (irregular) or a noncatastrophic (regular) form (Lee 1971; Stage 1973; Monserud 1976). As its name implies, catastrophic mortality is the result of usually unpredictable, massive disturbances such as fire, hurricanes, tornadoes, and insect or disease epidemics. Noncatastrophic mortality accompanies normal stand development. The models developed in this simulator predict only noncatastrophic mortality.

The following is a summary of the procedures used to obtain these models. Additional information is in appendix E.

Prior silvicultural and mensurational research in southwestern ponderosa pine showed that noncatastrophic mortality in uneven-aged stands is often correlated to diameter size, severity of cutting, and tree vigor as indicated by bark color (in other words, whether the tree is a blackjack or yellow pine). For even-aged stands, site and total stand basal area are significant in predicting total stand mortality.

I selected a nonlinear logistic function for modeling the proportion of trees in a diameter class succumbing to mortality. One reason for this is that the dependent variable of the logistic can be properly limited to a value between zero and one. Second, characterizing a dichotomous dependent variable such as mortality with the logistic function appears to produce a model with improved statistical properties when compared to ordinary, linear least squares models (Hamilton 1974; Hamilton and Edwards 1976).

To model the proportion of the trees in a diameter class dying in a growth period, the dependent variable was set to zero if the tree survived to the end of the growth period, and it was set to one if the tree died. The independent variables used were transformations of diameter class or stand attributes and incorporated not only the prior factors found to be correlated to mortality, but also factors hypothesized to be correlated. Included in this group were time-since-last-cutting and basal area growth predicted from the three completed models. Appropriate transformations were formed based upon both prior silvicultural and mensurational research findings, and an examination of the particular behavior of the logistic function.

The development of finished mortality models required the use of nonlinear regression and proceeded in three phases:

Phase 1.--For each vigor class, the data were separated into three sets: virgin uneven-aged, managed uneven-aged, and managed even-aged. Preliminary nonlinear regression runs, using the logistic function, were made to select those variables most highly correlated to mortality. The two best variables proved to be diameter class size squared and predicted basal area growth.

Phase 2.--The separated data sets were collapsed into two sets (uneven-aged, and even- and uneven-aged) by modeling the change in model parameters between the separated data sets as functions of time-since-last-cutting. An examination of a chi-square "goodness-of-fit" test for the best models indicated that the uneven-aged yellow pine equation (table 10) and the uneven-aged blackjack pine equations (tables 11 and 12) were satisfactory. The equations for even- and uneven-aged blackjack pine were not satisfactory.

Phase 3.--To resolve the problems with the even- and uneven-aged blackjack pine data set, another set of nonlinear regression runs was made. While these models were an improvement over the previous models, the chi-square values still indicated a significant difference between predicted and actual mortality (tables 13 and 14). An examination of the causes for mortality revealed that the even-aged data had incurred considerable snowbreak loss. Because I could not decide whether snowbreak should be treated as a catastrophic or noncatastrophic loss, these models were retained until the validation phase of the study could more closely examine the predictive capabilities of the equations.

Table 10.--*Chi-square test across diameter classes for best mortality equation for yellow pine*

Diameter class	Number of trees in class	Actual mortality	Predicted mortality	Chi-square value
4 - 11	83	2	1.97	0.0
12 - 13	129	3	2.71	0
14 - 15	145	3	2.79	0
16 - 17	229	5	4.17	.2
18 - 19	325	5	5.42	0
20 - 21	469	4	7.32	1.5
22 - 23	585	6	8.93	1.0
24 - 25	630	15	9.56	3.1
26 - 27	557	4	8.68	2.5
28 - 29	533	13	9.14	1.6
30 - 31	370	6	6.83	.1
32 - 33	260	10	5.73	3.2
34 - 35	154	3	3.84	.2
36 - 37	105	2	3.03	.4
38 - 39	47	2	1.59	.1
40+	30	0	1.25	1.3
			Chi-square statistic =	15.1

Table 11.--*Chi-square test across diameter classes for best mortality equation for uneven-aged blackjack pine using basal area growth equation for even- and uneven-aged blackjack pine*

Diameter class	Number of trees in class	Actual mortality	Predicted mortality	Chi-square value
4	4,416	11	19.11	3.4
5	4,066	18	17.37	0
6	3,322	18	14.11	1.1
7	2,629	17	11.33	2.8
8	2,021	8	8.81	.1
9	1,551	7	6.82	0
10	1,203	5	5.39	0
11	1,004	6	4.50	.5
12	893	3	4.01	.3
13	737	2	3.32	.5
14	686	1	3.19	1.5
15	666	6	3.19	2.5
16	685	4	3.36	.1
17	649	2	3.17	.4
18	610	1	3.06	1.4
19	473	0	2.33	2.3
20 - 21	908	6	4.83	.3
22 - 23	622	9	3.41	9.2
24 - 25	353	1	1.95	.5
26+	261	0	1.94	1.9
			Chi-square statistic =	28.9

Table 12.--*Chi-square test across diameter classes for best mortality equation for uneven-aged blackjack pine using basal area growth equation for uneven-aged blackjack pine only*

Diameter class	Number of trees in class	Actual mortality	Predicted mortality	Chi-square value
4	4,416	11	19.10	3.4
5	4,066	18	17.36	0
6	3,322	18	14.11	1.1
7	2,629	17	11.32	2.8
8	2,021	8	8.81	.1
9	1,551	7	6.81	0
10	1,203	5	5.39	0
11	1,004	6	4.50	.5
12	893	3	4.02	.3
13	737	2	3.32	.5
14	686	1	3.19	1.5
15	666	6	3.20	2.5
16	685	4	3.37	.1
17	649	2	3.19	.4
18	610	1	3.07	1.4
19	473	0	2.34	2.3
20 - 21	908	6	4.84	.3
22 - 23	633	9	3.41	9.2
24 - 25	353	1	1.94	.5
26+	261	0	1.93	1.9
			Chi-square statistic =	28.9

Table 13.--*Chi-square test across diameter classes for best mortality equation for even- and uneven-aged blackjack pine using basal area growth equations for even- and uneven-aged blackjack pine*

Diameter class	Number of trees in class	Actual mortality	Predicted mortality	Chi-square value
4	5,790	126	83.78	21.3
5	5,635	47	70.05	7.6
6	4,681	28	44.95	6.4
7	3,549	22	26.04	.6
8	2,540	8	14.34	2.8
9	1,792	7	8.56	.3
10	1,261	6	5.68	0
11	1,011	6	4.23	.7
12	893	3	3.54	.1
13	737	2	2.75	.2
14	686	1	2.53	.9
15	666	6	2.61	4.4
16	685	4	2.63	.7
17	649	2	2.52	.1
18	610	1	2.38	.8
19	473	0	1.96	2.0
20 - 21	908	6	4.07	.9
22 - 23	622	9	3.06	11.5
24 - 25	353	1	2.13	.6
26+	261	0	2.50	2.5
			Chi-square statistic =	64.3

Table 14.--*Chi-square test across diameter classes for best mortality equation for even- and uneven-aged blackjack pine using basal area growth equations for uneven-aged blackjack pine only*

Diameter class	Number of trees in class	Actual mortality	Predicted mortality	Chi-square value
4	5,790	126	83.56	21.6
5	5,635	47	70.76	8.0
6	4,681	28	44.83	6.3
7	3,549	22	25.58	.5
8	2,540	8	13.96	2.5
9	1,792	7	8.45	.2
10	1,261	6	5.71	0
11	1,011	6	4.30	.7
12	893	3	3.62	.1
13	737	2	2.82	.2
14	686	1	2.61	1.0
15	666	6	2.71	4.0
16	685	4	2.75	.6
17	649	2	2.63	.2
18	610	1	2.49	.9
19	473	0	2.05	2.1
20 - 21	908	6	4.15	.8
22 - 23	622	9	3.03	11.8
24 - 25	353	1	2.04	.5
26+	261	0	2.34	2.3
			Chi-square statistic =	64.3

The finished mortality equations are in table 15. Graphs for some of these equations, using representative values of the independent variables, are in figures 1, 2, and 3. Due to the similarity of the two basal area growth equations for blackjack pine, only one of the two mortality equations for uneven-aged blackjack pine and for even- and uneven-aged blackjack pine were plotted. In all of the equations, the mortality rate increases as D and MBA_2 increases and as BAG and TIME decreases. Caution should be taken when interpreting these graphs. Because of the interrelationships among D, BAG, and MBA_2, the actual mortality rate in a stand would not be represented by any single curve shown.

Conversion from Blackjack Pine to Yellow Pine

The classification of ponderosa pine as either blackjack pine or yellow pine is based on bark color. Young, vigorous growing trees have dark-colored bark while the mature or overmature, slow growing trees develop a yellow-colored bark (Harlow and Harrar 1958). As this study has shown, the trees in these two "vigor" classes display different growth and mortality rates and, as a result, an improvement in modeling growth and mortality can be expected if the two vigor classes are treated separately. By treating them separately, however, it then becomes necessary to develop a model predicting the conversion from blackjack pine to yellow pine.

Table 15.--*Finished mortality equations*

$$PM = (1.0 + e^{-X})^{-1}$$

where

PM = proportion of trees in a diameter class dying in the next 5-year period

For uneven-aged yellow pine:

$$X = -3.612474 - 1.763910*YPBAG - 9.281652*A_1*YPBAG + 5.597303E\text{-}04*D^2$$

For uneven-aged blackjack pine:

$$X = -5.372649 + 1.270862E\text{-}03*D^2 - 9.599418*A_1*BJBAG1 \quad \text{or}$$

$$X = -5.372991 + 1.271523E\text{-}03*D^2 - 9.388071*A_1*BJBAG2$$

For even- and uneven-aged blackjack pine:

$$X = -4.670214 - 18.54357*BJBAG1 - 7.948837E\text{-}03*TIME + 1.870818*(TIME)^{-2} + 1.370731E\text{-}02*MBA_2 + 4.382886E\text{-}03*D^2 \quad \text{or}$$

$$X = -4.725930 - 17.46734*BJBAG2 - 7.256918E\text{-}03*TIME + 1.883188*(TIME)^{-2} + 1.423927E\text{-}02*MBA_2 + 4.168657E\text{-}03*D^2$$

D = diameter class size

YPBAG = predicted yellow pine basal area growth

BJBAG1 = predicted even- and uneven-aged blackjack pine basal area growth

BJBAG2 = predicted uneven-aged blackjack pine basal area growth

TIME = number of 5-year periods since last cutting

A_1, A_2, A_3 = sigmoidal transforms of TIME. See table 5 for equations.

MBA_2 = basal area per acre in the given diameter class plus the two adjoining larger and smaller diameter classes

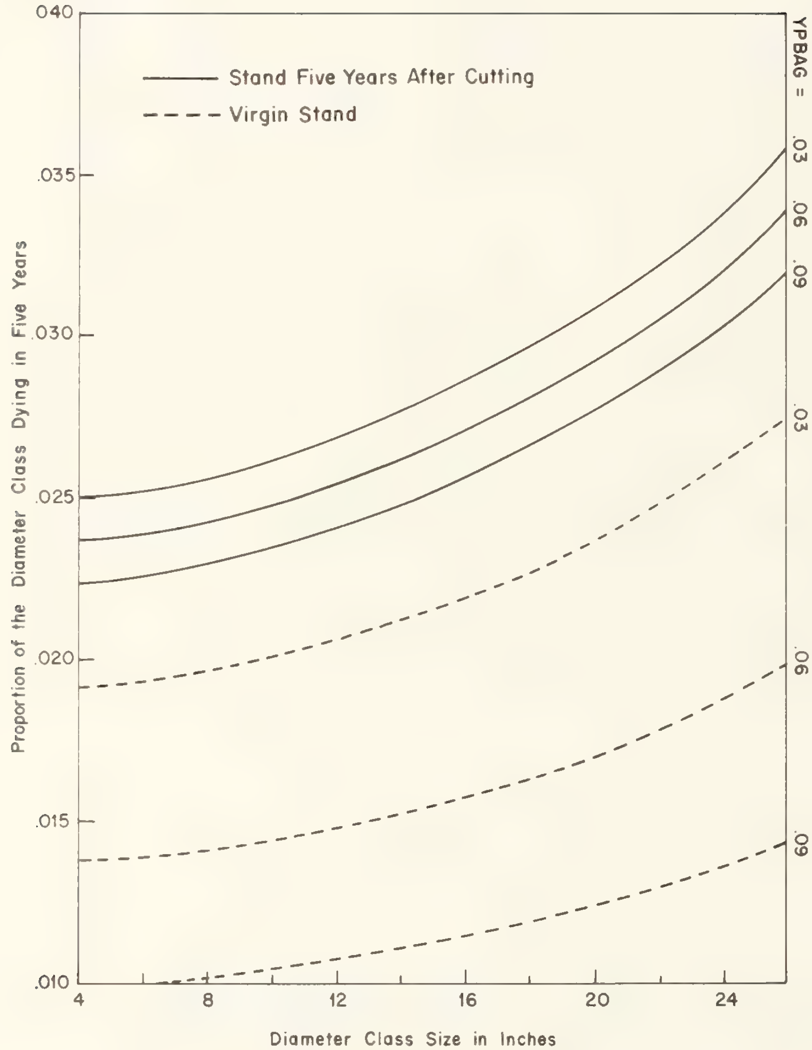

Figure 1.--Predicted 5-year mortality rate for yellow pine diameter class size and basal area growth (YPBAG).

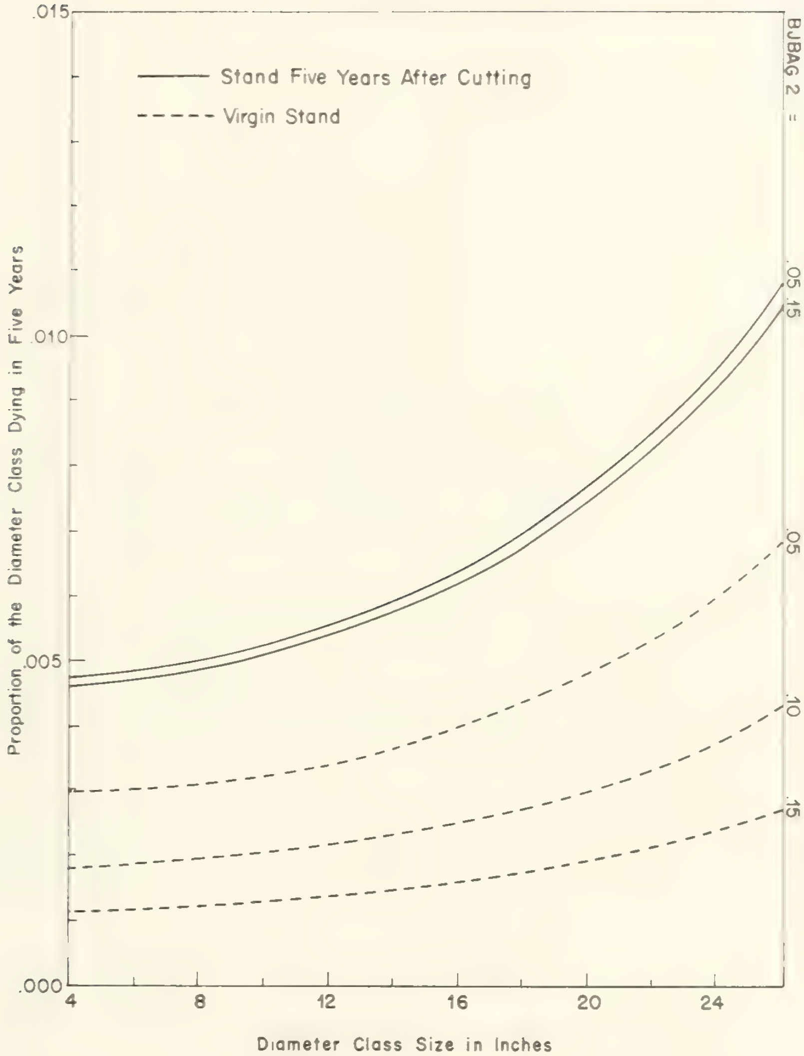

Figure 2.--Predicted 5-year mortality rate for uneven-aged blackjack pine by diameter class size and uneven-aged basal area growth (BJBAG2).

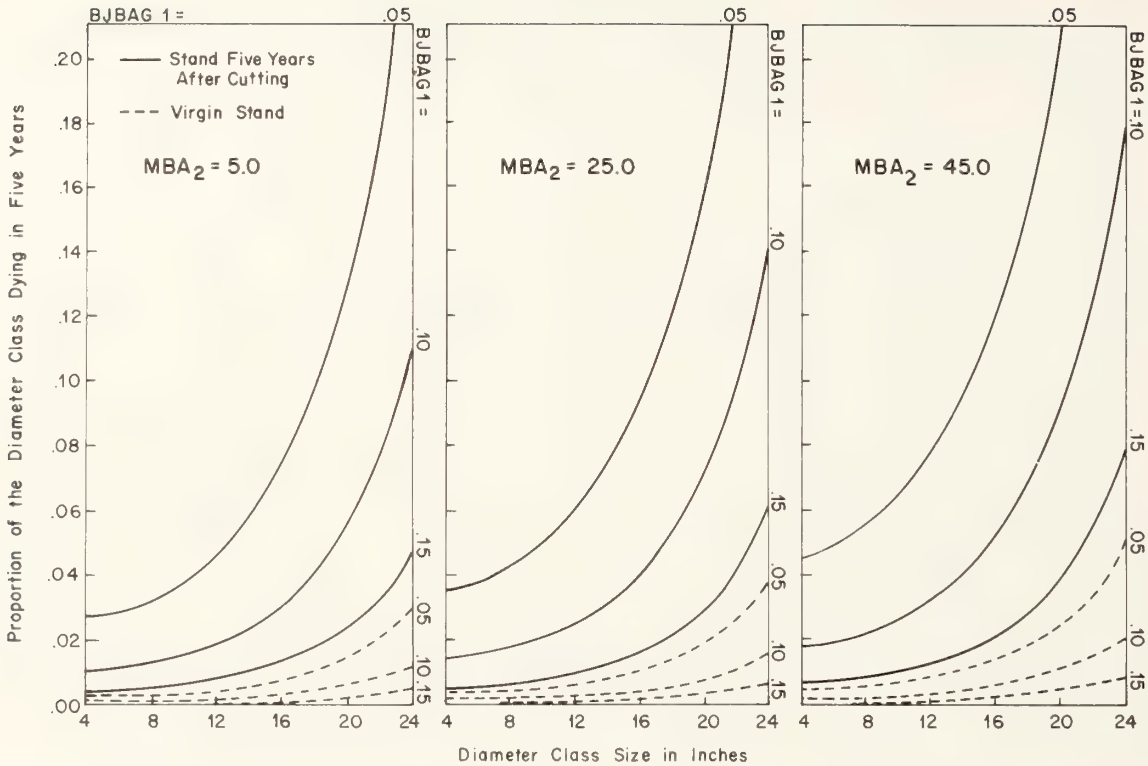

Figure 3.--Predicted 5-year mortality rate for even- and uneven-aged blackjack pine by diameter size class, MBA_2 class, and even- and uneven-aged blackjack pine basal area growth (BJBAG1).

During data collection, all trees were initially categorized by vigor class, and then those trees not cut on a plot were reclassified every 20 years (that is, at the start of every fifth growth period). An examination of the data available for modeling conversion indicated the following problems:

1. While growth, mortality, and ingrowth could be predicted every growth period, conversion from blackjack to yellow pine could only be predicted after every fourth growth period. This might cause the number of trees in each vigor class to change abruptly after every fourth growth period.

2. Cutting of blackjack trees between the reclassification times negates the utility of the plot for predicting conversion because of the uncertainty of whether those trees cut would have converted or not. The plot and classification period combination without cutting are the first classification period of plot 62 and both periods of plot 61.

3. The conversion from a blackjack pine to a yellow pine is a slow but continual process, and therefore, the discrete classification of a tree as either blackjack or yellow pine is extremely subjective. This subjectivity can become a problem on long-term studies because the classification of vigor classes is done by different people with individualized critera for classification. An examination of the conversion rates for the first 20 years on plots 61 and 62, compared to the conversion rate on plot 61 in the second 20 years, shows that the rates differ considerably between the two classification periods. While it cannot be proved, it is likely this difference is due in part to the different individuals performing the classifications.

Based on these problems, I decided to use only the data from plot 61 (both classification periods) to develop the conversion rate model. The data for the first classification period on plot 62 were eliminated since including that data would unduly weight the resulting model towards the low conversion rate of the first classification period. By using just the data from plot 61, an approximately equal weight is given to both periods.

The usage of data from the second classification period on plot 61 does pose an additional problem. During the second classification period, only trees 7.6 inches and larger were measured. All stand density variables, therefore, must be expressed in trees 7.6 inches and larger. In addition, the usage of predicted basal area growth is precluded because of the equation's dependency upon knowing stand basal area of all trees 3.6 inches and larger.

The following is a summary of the process used to develop the final conversion model. Further details are in appendix F.

I anticipated the factors influencing the conversion rate would be the same as those that might influence tree vigor, as expressed by growth. Previously it was shown these factors include measures of productivity, diameter class size, and stand density. Keeping in mind both these factors and the restriction that only stand attributes for trees 7.6 inches and larger could be used, a number of variables were defined for use as independent variables.

As with mortality, I decided to express the rate as a proportion of the trees in a diameter class converting from blackjack to yellow pine and to model this rate using the nonlinear logistic function. Therefore, I assigned the dependent variable a value of one if the blackjack tree converted to yellow pine, and a value of zero if it did not.

Using these variables, a number of nonlinear regression runs were made, and chi-square "goodness-of-fit" values were examined. Predictions from all models displayed significant differences from the actual values. Examination of the models and data revealed that the greatest misfit was due to data from a single subplot. Therefore, I decided to strengthen the data base by adding to it the data from those subplots originally eliminated because of the paved highway. A second set of nonlinear regression runs was made, but again the resulting models displayed poor fit in the same diameter class.

Because tree vigor is such a subjective classification, the apparently atypical subplot was eliminated and new equations determined. The resulting model's predictions were not significantly different from the actual values (see table 16). To test this model further, chi-square values were also computed for the data reserved for validation. The result indicated a significant difference between the model predictions and the actual data, and the cause was traced to another subplot with an unusual conversion rate in the same diameter class. I could not determine why the two subplots exhibited such high conversion in the same diameter class, but the elimination of the second subplot from the data did produce an insignificant chi-square value for the validation data.

The final conversion model is:

$$PrCON = (1.0 + e^{-X})^{-1}$$

where

$$X = -3.777269 + 0.5012605D - 6.597465E\text{-}03D^2 - 0.0982176S$$

PrCON = proportion of trees in a diameter class converting from blackjack pine to yellow pine

D = diameter class size

S = Minor's site index

A plot of this equation is in figure 4. To use this model, each independent variable is assigned the value existing for the diameter class at the start of the growth period prior to reclassification. Therefore, at the start of every fourth growth period, a prediction is made as to how many trees will convert by the start of the fifth growth period.

Table 16.--*Chi-square test across diameter classes for final conversion model, developed using the expanded data set, without data from subplot 13*

Diameter class	Number of trees in class	Actual conversion	Predicted conversion	Chi-square value
4 - 9	1,486	0.00	1.33	1.3
10 - 11	317	0	.94	.9
12 - 13	228	1.00	1.39	.1
14	103	4.00	1.06	8.1
15	99	1.00	1.36	.1
16	91	6.00	1.75	10.3
17	91	1.00	2.43	.8
18	93	4.00	2.81	.5
19	100	4.00	4.03	0
20	87	2.00	4.70	1.6
21	79	5.00	4.87	0
22	74	4.00	5.29	.3
23	64	8.00	6.17	.5
24	47	1.00	5.21	3.4
25	42	6.00	5.69	0
26	32	4.00	5.13	.2
27	24	5.00	4.13	.2
28	15	4.00	3.58	.1
29+	19	5.00	4.32	.1
			Chi-square statistic =	28.7

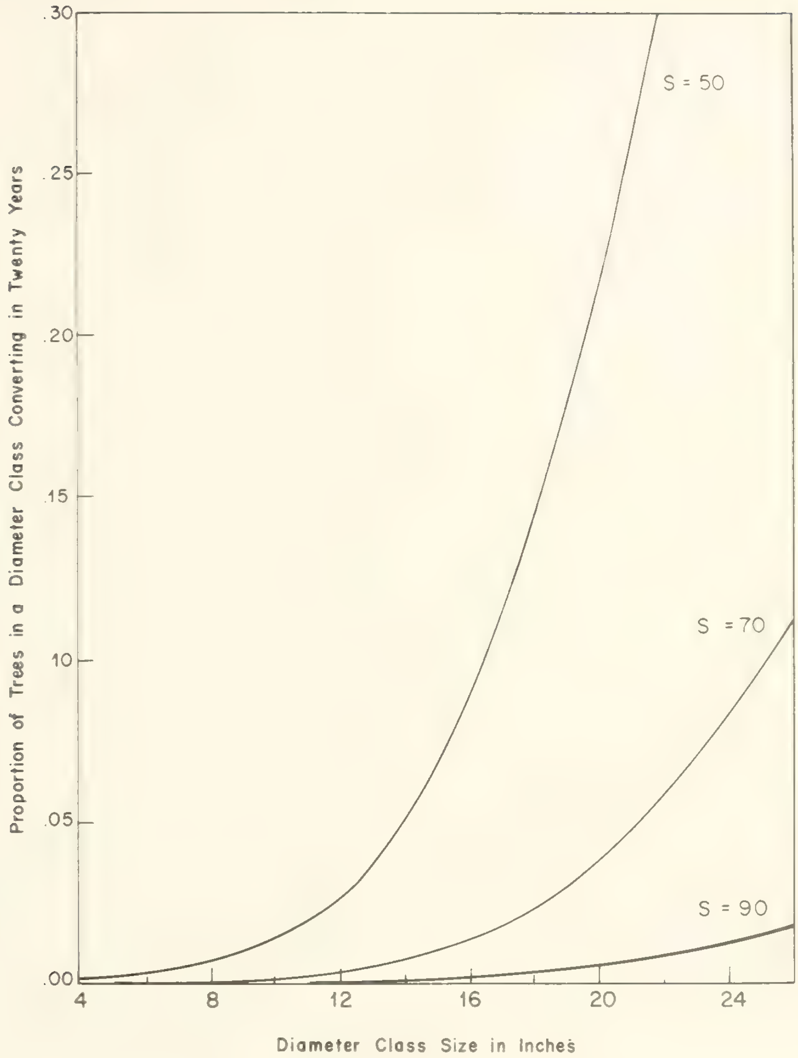

Figure 4.--Predicted conversion from blackjack pine to yellow pine by diameter class size and site index (S).

Recruitment

The requirements for adequate natural regeneration in the southwestern ponderosa pine are the presence of large, vigorous trees well dispersed over the area, a good seed crop, adequate rainfall in two consecutive years, and low level of competitive vegetation (see appendix G for details). This combination of conditions occurs at erratic invervals and, as a result, adequate natural regeneration is erratic. For example, the last excellent regeneration period occurred in 1918-1919 (Pearson 1950; Meagher 1950; and Schubert 1974). Cooper (1960) has concluded that erratic natural regeneration is a major reason for the "unbalanced" size structure of the ponderosa pine forests of the Southwest.

Ideally, a natural regeneration model should take all of these factors into consideration as components of it (such as the one described by Leak and Graber 1976). Unfortunately, the data available in this study preclude the development of a regeneration model. This was unfortunate because the greatest distinction between even- and uneven-aged stand development is the method of regeneration. The even-aged stand usually starts as a surge of regeneration over a relatively short period of time (20 years or less). Next is a stage where little or no regeneration takes place. Finally, some regeneration may again occur (depending upon the species and site characteristics) as the stand becomes mature (or overmature) and develops a more open structure. On the other hand, the uneven-aged stand is usually characterized by a more or less continuous flow of regeneration into the stand.

Because of the data restrictions, the only "recruitment" model possible at this time is an ingrowth model. In this study, ingrowth is defined as the number of new trees growing into the 4- or 5-inch diameter classes during a growth period. Ingrowth, therefore, includes not only those factors influencing the amount of established seedlings, but also includes those factors influencing the growth and mortality rate of seedlings as they advance to ingrowth size. Conditions providing for a large, established seedling crop and allowing favorable seedling growth and low mortality will ultimately result in a large ingrowth rate at some point in the future.

For uneven-aged stands, ingrowth can be expected at any time, but in even-aged stands, ingrowth would only be expected during the early, and possibly the late, stand development phase. Unfortunately, the Taylor Woods even-aged data represent only stands in relatively early stand development. As a result, any ingrowth model developed using just the Taylor Woods data would be applicable only to stands in the same stand development phase. Such a restriction would give the model little utility for predicting even-aged stand development through all phases of development. Therefore, it was decided that the ingrowth model would be developed only for uneven-aged stands. An alternative method for even-aged stands by which use of the basal area and mortality models are extended down into the 1-, 2-, and 3-inch diameter classes is proposed and tested in the validation phase.

Two equations were used to model uneven-aged ingrowth. The first predicts the total number of ingrowth trees. The second predicts the proportion of the total ingrowth trees that will grow through the 4-inch diameter class and into the 5-inch diameter class during the growth period.

TOTAL INGROWTH

After examining prior silvicultural and mensurational findings, I hypothesized that the factors possibly influencing the rate of ingrowth for uneven-aged stands included potential of the stand to produce cones, level and structure of competition, site index, mean stand diameter, and quadratic mean stand diameter. Based on these factors, appropriate independent variables were defined. The following is a summary of the process used to develop the total ingrowth model. Further details are in appendix G.

Two factors influenced the development of a strategy for modeling total ingrowth. First, while a nonlinear total ingrowth model is most likely, it was not known whether the residuals about the model are multiplicative (implying that taking the log of the model could linearize the data) or additive. Second, the form and specification of the applicable independent variables in the final ingrowth model had not been rigorously defined. Therefore, a good deal of screening was necessary before the final model could be determined. Given these conditions, the following strategy was used to develop a total ingrowth model:

Phase 1.--The total ingrowth data were separated into two sets: virgin uneven-aged and managed uneven-aged. For each set, a preliminary all-combinations screening run using linear regression was made on the variables linearized by applying logarithms, and selected models were chosen for further analysis.

Phase 2.--A second screening run combined the two basic data sets through use of the time-since-last-cutting transforms. Because the sign on site index was not considered reasonable, a new dependent variable was formed by dividing total ingrowth by site index and taking the log of the quotient. A new set of screening runs was then made and the final independent variables were chosen.

Phase 3.--An examination of the residuals of the preceding model indicated that the error structure of total ingrowth divided by site index might not be best represented by a log model. Therefore, the antilog of the model was fitted with a linear, least squares slope correction. An examination of these residuals revealed that the error structure of total ingrowth divided by site index was better represented by a weighted, nonlinear model with an additive error.

Phase 4.--Weights were then developed and used in a weighted, nonlinear regression program to determine the following, final weighted nonlinear model:

$$\text{Total ingrowth} = d_0 S(BACL_1)^{d_1}(1.0+BACL_2)^{d_2} e^{(d_3 BA^{d_4} + d_5 A_1 BA^{d_4})}$$

where

$d_0 = 0.48630258$ $\quad$ $d_3 = -3.97721395E-06$
$d_1 = 0.65949418$ $\quad$ $d_4 = 2.7667388$
$d_3 = -0.49280582$ $\quad$ $d_5 = -1.33299951E-06$

where

S = Minor's (1964) site index
BA = total stand basal area per acre in square feet
$BACL_1$ = basal area per acre in the 4- through 6-inch diameter classes
$BACL_2$ = basal area per acre in the 7^+-inch diameter classes.

The weighted, mean square error for this model was 0.78244674. The coefficient of determination, R^2, was 0.8285 for 201 observations. See figure 5 for a graph of this function.

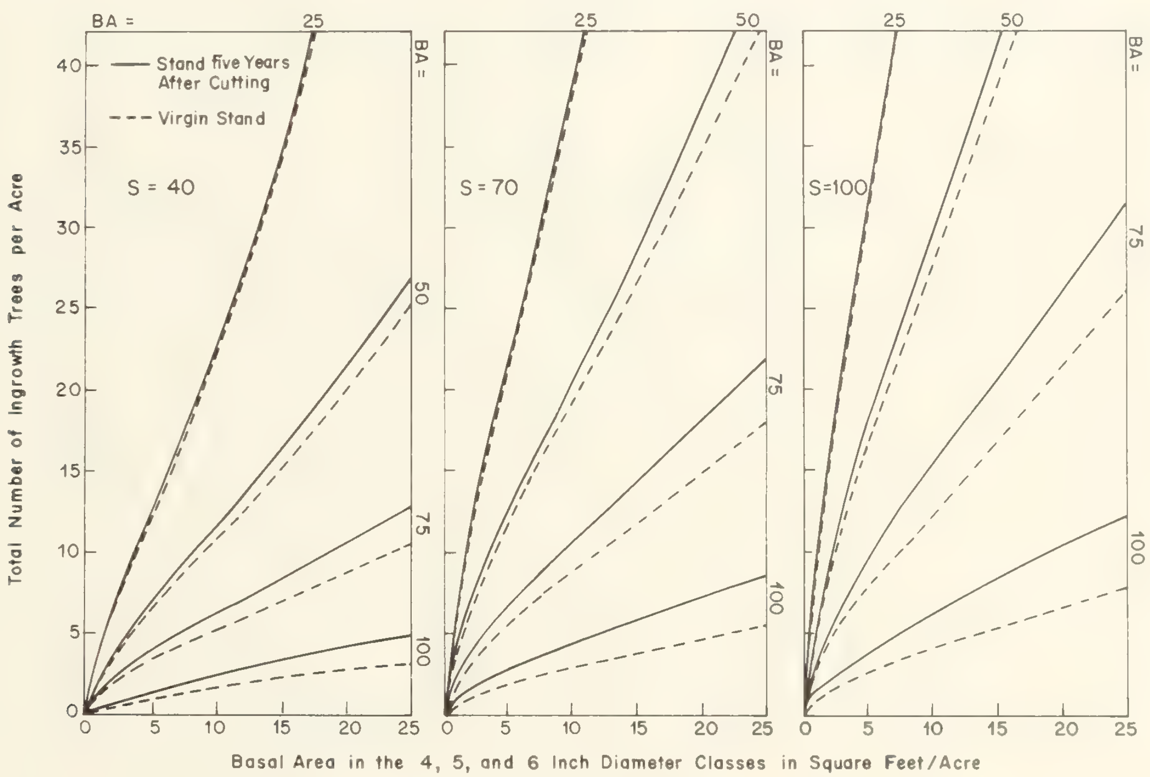

Figure 5.--Predicted total number of ingrowth trees by site index classes (S), total basal area classes (BA), and the basal area in the 4-, 5-, and 6-inch diameter classes.

THROUGH-GROWTH

Through-growth is defined as the proportion of the total ingrowth that will grow through the 4-inch diameter class and into the 5-inch class, in the same growth period. It is assumed that no trees will grow through to the 6-inch class.

As with the mortality and conversion models, I modeled through-growth using the nonlinear logistic function. Independent variables tried in this weighted, nonlinear analysis included predicted basal area growth in the 4-inch diameter class, predicted total ingrowth, and time-since-last-cutting. The finished models were selected after fitting and closely examining a number of alternative models. The regression coefficients for these models are in table 17 and plots of the models in figures 6 and 7.

Table 17.--*Final through-growth equations*

$$\text{Through-growth} = (1.0 + e^{-x})^{-1}$$

where

Through-growth = proportion of the total number of ingrowth trees that through grow into the 5-inch diameter class

$$x = \begin{cases} -2.636336 + 28.73438*\text{BJBAG1-4''} - 0.03419626*A_1*\text{TINGRO} \\ \text{or} \\ -2.490266 + 24.95862*\text{BJBAG2-4''} - 0.03440381*A_1*\text{TINGRO} \end{cases}$$

BJBAG1-4" = predicted basal area growth of the 4-inch diameter class using the even- and uneven-aged blackjack pine equation

BJBAG2-4" = predicted basal area growth of the 4-inch diameter class using the uneven-aged blackjack pine equation

TINGRO = predicted total number of ingrowth trees

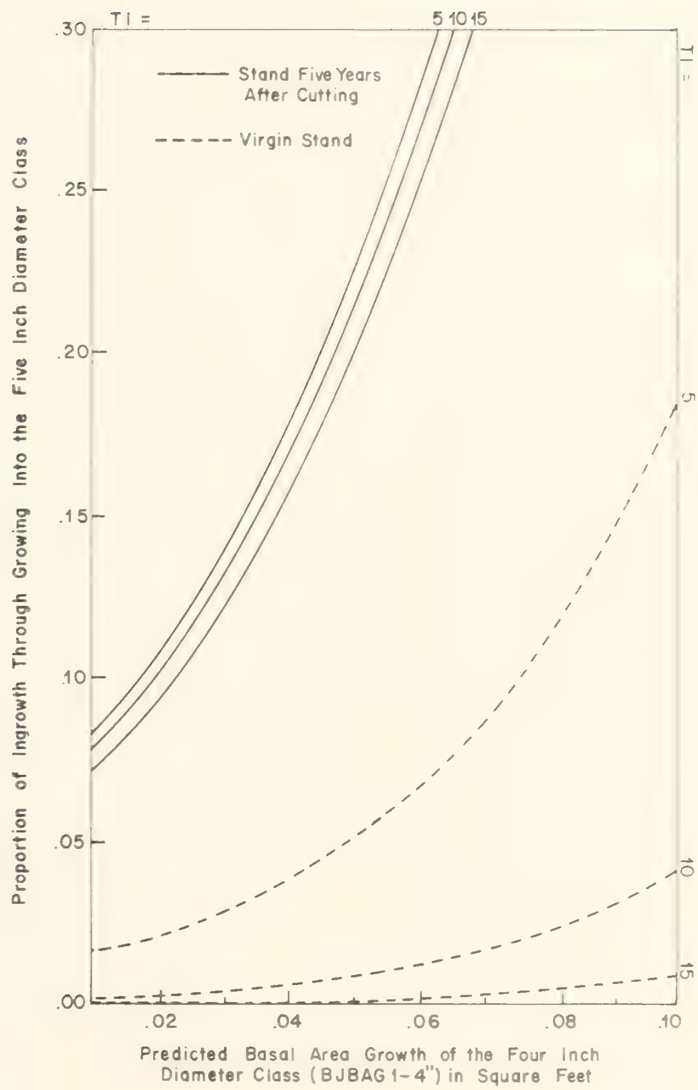

Figure 6.--Predicted through-growth by BJBAG1-4" and total ingrowth (TI).

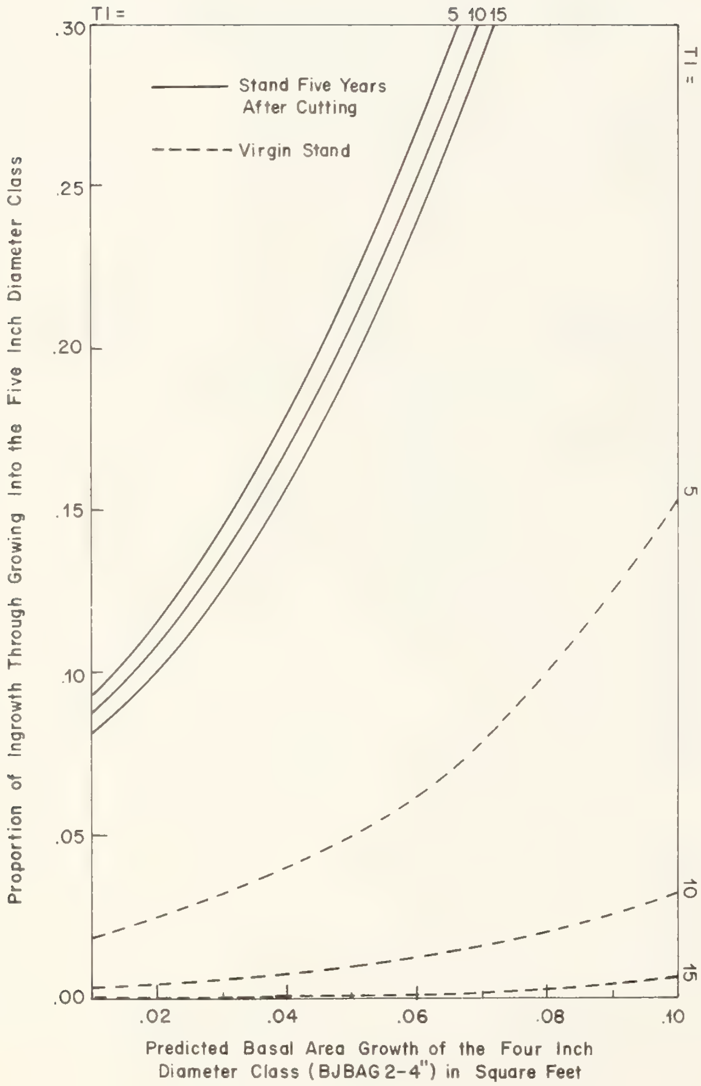

Figure 7.--Predicted through-growth by BJBAG2-4" and total ingrowth (TI).

Height and Cubic-foot Volume Equations

The lack of height-growth data necessitated developing alternative height equations for assessing product potential as predicted by cubic-foot volume equations. Two separate equations were developed for even-aged and for uneven-aged stands.

EVEN-AGED HEIGHT EQUATION

Minor (1964) developed an equation for even-aged blackjack stands in northern Arizona that predicts heights of dominant trees as a function of site index and stand age. Plots of average diameter class heights over diameter class size for the Taylor Woods height data indicated that average diameter class height was below the height predicted by Minor's (1964) equation for the smallest diameter classes and increased, asymptotically, to Minor's (1964) height prediction in the largest diameter classes. Using the procedure outlined in appendix H, the following equation was developed for use in even-aged, blackjack stands:

$$H_D = MH - 0.88015037(MH-4.5)(1.0 - \frac{D}{MD})^{1.35}$$

where

H_D = average height of the D*th* diameter class

D = diameter class size

MH = maximum height as predicted by Minor's (1964) equation

$= S - 1.4003\ (\sqrt{A} - 10) + 0.1559(S)(\sqrt{A} - 10)$

S = Minor's (1964) site index

A = breast height stand age, $20 \leq A \leq 140$

DM = maximum diameter class size that the stand has achieved in its development.

UNEVEN-AGED HEIGHT EQUATION

Because stand age is meaningless in uneven-aged stands, the foregoing even-aged equation cannot be applied to uneven-aged stands. As an alternative, an equation that predicts average diameter class height as a function of site index and diameter class size was developed using the procedure described in appendix H. The equation is:

$$H_D = b_0 + b_1 \cdot S^{b_2} \cdot e^{b_2 (D+35)^{-2}}$$

where

H_D = average height of the D*th* diameter class

D = diameter class site

S = Minor's (1964) site index

$b_0 = 4.5$

$b_1 = 13.178649$

$b_2 = 0.71631005$

$b_3 = -4221.6528$

This equation is for both blackjack and yellow pine. Insufficient data precluded separate equations.

TOTAL STEM CUBIC-FOOT VOLUME EQUATIONS

Existing equations or data for developing new equations to predict total stem cubic foot volume in 1-inch diameter classes were not available for northern Arizona. As an alternative, I used tree volume equations developed by Hann and Bare (1978) for yellow and blackjack pine on the Coconino National Forest of northern Arizona. While application of an individual tree volume equation to diameter class attributes could produce biased volume estimates, I felt the smallness of 1-inch classes would minimize the potential problem.

Structure of Simulator

The four models--upgrowth, mortality, conversion from blackjack pine to yellow pine, and ingrowth--interact in the following fashion:

1. Present number of trees in each diameter and vigor class provide the starting point.

2. Mortality is computed and subtracted from each class to give the number of trees expected to survive to the end of the growth period.

3. Using diameter class growth of the survivors and the appropriate within diameter class distribution, the number of trees moving to various diameter classes is determined.

4. At the start of each fourth growth period, the blackjack-to-yellow pine conversion rates for survivors are calculated; otherwise, the conversion rates are set to zero. The conversion rates and upgrowth information then are used to allocate the survivors to the appropriate "future" diameter and vigor classes.

5. Ingrowth for the next 5-year period is computed and added to the 4- and 5-inch classes.

6. Removals due to cutting are made from each diameter class. Cutting, therefore, presumably occurs at the end of the growth period. This step completes the "future" diameter distribution.

7. This distribution then becomes the present diameter distribution, and the cycle starts again. A system interaction chart is found in figure 8.

VALIDATION OF SIMULATOR

The problem of validating a simulator has troubled modelers for years. Shannon (1975) identified and described the philosophies of three approaches. "Rationalism holds that a model is simply a system of logical deductions from a set of premises, which may or may not be open to empirical verification or appeal to objective experience" (page 212). "Empiricism refuses to admit any premises or assumptions that cannot be verified independently by experiment or analysis of empirical data" (page 214). Finally, absolute pragmatism holds that a model is designed to meet a need, and if the model succeeds in meeting the need, then it has been validated.

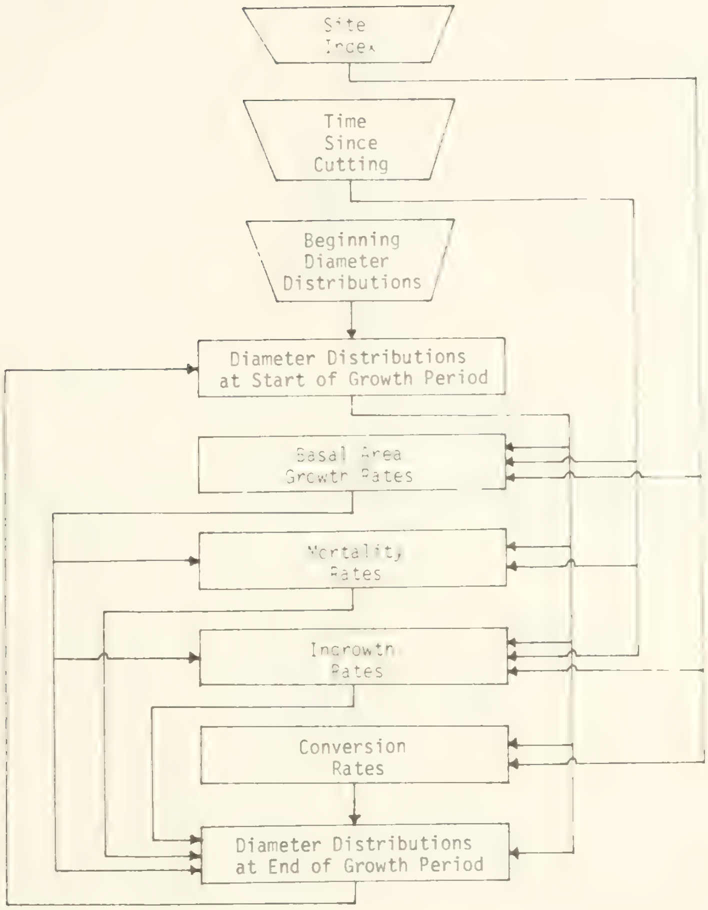

Figure 8.--System interaction chart.

People interested in validation have come to realize that none of the three philosophies fully meets validation needs individually or is applicable to all situations. As a result, a utilitarian approach has evolved. Shannon defines the three steps of this approach:

> The first stage is to seek face validity of the internal structure of the model based upon a priori knowledge, past research, and existing theory (page 215).
> The second stage is also concerned with the validation of the internal structure of the model, and consists in empirically testing, whenever possible, the hypothesis used (page 215).
> The third stage attempts vigorously to verify the model's ability to predict the behavior of the real world system (page 216).

Many applications to forestry require the simulator to be both a prediction (or "prognosis") tool and a technique for examining the dynamics of the system. This requires the simulator to be accurate not only in a behavioral sense but also quantitatively. Many of the system components are slow to react and, coupled with the general long-range aspects of forestry, require long simulation runs to fully monitor true system behavior. With these special characteristics in mind, I slightly modified and expanded upon the preceding utilitarian philosophy to produce the following tentative "rules of validation."

1. The simulator components should have a sound theoretical basis that uses as much experimental evidence and expert knowledge as possible. This would include identifying the significant system components, the factors that can influence these components, and applicable forms and techniques for modeling them.

2. Correct statistical and modeling techniques should be used to develop estimates of the parameters of the basic components.

3. Where possible, the estimated parameters and model forms of these components should be validated with a data source independent from that first used to estimate the parameters.

4. Like its components, the validation of the simulator should also be made on a data set independent of those used to derive the simulator. This is necessary only if the simulator is to be applied to studies other than those used in its development (which is the case most of the time in forestry applications). This or these independent data set(s) should also have a form compatible with the underlying structure of the simulator and should cover as wide a range of conditions as possible.

5. Validation should occur on the basic attributes that the simulator produces. For example, if the simulator is an individual tree/distance dependent one, then validation should occur on the tree level attributes rather than on stand level attributes. Aggregation tends to cover up inconsistencies.

6. Validation should occur over long enough time to allow valid tests (or comparisons) of long- as well as short-term responses and of subtle changes that take a long time to manifest themselves.

7. Where possible, some type of "statistical" test should be used to aid in quantifying validity.

Applying the Rules

The first three rules are normally applied during the model building process, while the last four rules are applicable to the final simulator. To incorporate the first two rules in this study, the following steps were taken in the model building process: (1) an exhaustive examination of the literature to identify major components of the simulator, the factors influencing these components, and applicable model forms; (2) an insistence that the various equations used to model these components meet the expected model forms and behave reasonably across the expected input data range; (3) identification of the correct statistical tools to model the components; and (4) where possible, a test of the assumption of both the models and statistical tools.

I decided, however, to develop a validation process that combined rule 3 with the last four rules so I could evaluate each component by how it influences the prediction of future diameter distributions, and by the necessity of saving time and conserving monetary resources. To combine rule 3 with the last four rules, it was necessary to make four different validation runs. The first run used actual values for all components except upgrowth; that is, only upgrowth was predicted. The second run used predicted upgrowth and mortality. The third run used predicted upgrowth, mortality, and conversion. The final run predicted all components (upgrowth, mortality, ingrowth, and conversion). In this way, the effect of adding each predicted component to the simulator could be evaluated by how the simulator predicts the basic attribute of interest, the future diameter distributions (rule 5).

Because two blackjack pine equations were developed (one with the even-aged data and one without), both equations were tested on the first set of validation runs. The equation that performed best was the final blackjack pine growth equation. A similar approach was used on the second set of runs to evaluate various mortality models.

Validation runs were made on the data reserved for that purpose (rule 4). Additional work was needed to make diameter distribution predictions past 1940. The difference between the pre- and post-1940 periods was the number of diameter classes that could be compared (for pre-1940, all diameter classes 4 inches and greater could be compared while, after 1940, only diameter classes 8 inches and greater could be compared).

The choice of what statistics and tests were appropriate for comparing predicted to the actual diameter distributions (rule 7) proved to be difficult. After reviewing or trying a

number of alternatives, none of them proved either applicable or useful (see appendix I for further details on tests examined or tried). A test not examined in this study but which could prove useful in future studies is described by Freese (1960).

The lack of a useful statistical test, therefore, necessitated a subjective evaluation of the validation runs. This decision led to an additional problem: how to conveniently summarize, for reporting and comparative purposes, the large amount of detailed individual subplot validation information produced in validating for the uneven-aged plots. After consideration, I concluded that two statistics adequately reflected the average behavior exhibited by just the validation subplots. These statistics were computed by first averaging across all the plot's subplots the predicted and actual diameter distribution for each growth period, combining blackjack and yellow pine in the process. Next, the mean difference between average actual and average predicted diameter distributions (the first statistic) and the variance of these differences (the second statistic) were computed across diameter classes for each plot and growth period.

Use of averages incorporating both the model building and the validation data sets would appear to violate the fourth rule of validation (that is, the validation data should be independent of the model building data). But the apparent violation is not serious because much of the basic validation information produced and examined (but not reported here) for decision-making pertained to just the subplots from the validation data set.

For the even-aged validation data, the mean difference across diameter classes, between the actual and predicted diameter distribution for each individual plot and growth period and the variance of these differences were used as summarization statistics. Statistics produced from average diameter distributions across all plots are meaningless for the even-aged data set.

Results of Validation

PREDICTED UPGROWTH; ACTUAL MORTALITY, CONVERSION, INGROWTH

The first set of validation runs used predicted upgrowth and actual mortality, cutting, conversion, and ingrowth rates to assess the predictive capabilities of the various upgrowth models.

In addition to the two blackjack pine basal area growth equations tested, two methods for applying the equations were also tested. One method used the average basal area growth for the diameter class to advance the trees, and the other divided the diameter class into three more classes and then used estimates for the lower, middle, and upper basal area growths of the diameter class to advance each third separately. The combination of basal area growth equations and advancement techniques resulted in the evaluation of four different upgrowth models.

For simplicity, the actual mortality, conversion, and cutting rates were expressed as proportions rather than number of trees. This approach has the advantage of eliminating the problems associated with removals being larger than the number of trees in a diameter class. The method, however, exaggerates differences between predicted and actual upgrowth because any difference between the resulting predicted and actual diameter distributions will result in different numbers of trees being removed due to mortality, cutting, or conversion.

Examining these runs revealed that, on the uneven-aged plots, the blackjack pine basal area growth equation developed with both even- and uneven-aged data (BJBAG1) behaved the same as the equation developed with just the uneven-aged data (BJBAG2). As a result, BJBAG1 was adopted as the final basal area growth equation for blackjack pine.

In addition, the upgrowth rate appeared too high. This led to an immediate suspicion of the proposed correction for log bias because of the magnitude of that correction (a 35 percent increase for blackjack pine and 140 percent increase for yellow pine) and because the justification for that correction was not too firm. Therefore, I decided to try two additional

correction schemes: no correction, and a correction of $ln(\overline{Y}/\overline{\hat{Y}})$, where $\overline{Y}$ is mean basal area growth and $\overline{\hat{Y}}$ is mean predicted basal area growth with no correction for log bias. I chose the latter because the correction was about halfway between the original correction and zero correction.

Six validation runs were made on each plot, each run representing a different log correction and one of the two methods for applying the equations (that is, average basal area growth or basal area growth in thirds). From these runs I concluded that the best log bias correction was no correction, and the best method for applying the equations was to divide the diameter classes into thirds and project each third.

The conclusion that a zero (when added to intercept terms of the log model) correction is best of the three methods should not be construed to mean that a zero correction is the best of all possible ones. A more likely interpretation is that the correction should be smaller than the two values tested. Obviously, the correction method posed in appendix D does not produce appropriate corrections for log bias. Until a more theoretical basis for correcting log bias in the nonnormal case is developed, the most reasonable approach may be to not correct the equations directly for log bias, but rather, to build in a calibrating routine that will correct for log bias, site differences, and so forth.

Table 18 aids comparison of the effects of adding predictive components upon accuracy (as measured by average differences between predicted and actual number of trees in a diameter class) and precision (as measured by the variance of the differences). The first two columns of values provide the mean difference and the variance of differences for each plot and growth period (since initialization) for the set of runs using the final predicted upgrowth model and actual mortality, conversion, and ingrowth. If a model predicted upgrowth perfectly, the resulting values would be zero. While the values do differ from zero, their magnitudes indicate that upgrowth predictions are reasonably good.

As expected, results also indicate that, as the number of growth periods from initialization increases, the mean differences and the variance of the differences also increase.

PREDICTED UPGROWTH AND MORTALITY; ACTUAL CONVERSION AND INGROWTH

I tested two blackjack pine mortality equations in this set of runs: one equation was developed using the uneven-aged data, and the other using both the even- and uneven-aged data. I developed the single yellow pine equation using uneven-aged data.

Analysis of the runs showed the even- and uneven-aged blackjack pine equation evidently inferior to the uneven-aged blackjack pine equation for predicting uneven-aged mortality. The former equation also did not predict even-aged mortality well. This finding was not surprising because of the high level of snowbreak loss on the even-aged plots.

A different approach for predicting even-aged mortality was tried. I reclassified the snowbreak mortality as a "cutting" loss (that is, not endemic), and predicted the remaining mortality using the uneven-aged mortality equations. The result was greatly improved runs over those using even- and uneven-aged mortality equations.

This finding indicates that if catastrophic losses (such as snowbreak, fire, insects, or disease) are treated in the same fashion as cutting losses, endemic losses in even- and uneven-aged stands could be predicted using the same mortality equations, if the equations are developed appropriately.

The third and fourth columns of table 18 provide the mean difference and the variance of differences for the runs using the final upgrowth and mortality models and actual conversion and ingrowth. A comparison with the values derived from the previously discussed set of runs indicates that adding the mortality functions has not greatly affected accuracy and precision.

Table 18.--*Mean difference and variance of differences between average actual and average predicted number of trees per diameter class for a specified plot, growth period, and simulation type*

Plot	Growth period	Predicted upgrowth		Predicted upgrowth and mortality		Predicted upgrowth, mortality, conversion		All components predicted	
		Mean difference	Variance of difference	Mean difference	Variance of difference	Mean difference	Variance of difference	Mean difference	Variance of difference
61	1	0.0028	0.0151	0.0135	0.0153	0.0135	0.0153	0.0283	0.0138
	2	.0027	.0495	.0183	.0486	.0183	.0486	.0568	.0371
	3	.0021	.0615	.0231	.0640	.0231	.0640	.0683	.0563
	4	.0092	.0101	.0263	.0118	.0263	.0118	.0274	.0107
	5	.0082	.0102	.0279	.0126	.0282	.0126	.0366	.0104
	6	.0124	.0244	.0268	.0255	.0271	.0255	.0474	.0419
	7	.0126	.0327	.0295	.0366	.0300	.0365	.0597	.0573
	8	.0261	.0395	.0450	.0459	.0455	.0457	.0800	.0713
	10	.1000	.4436	.1239	.4596	.1242	.4581	.1600	.4667
62	1	.0000	.0093	.0080	.0094	.0080	.0094	-.0027	.0294
	2	-.0007	.0166	.0102	.0162	.0102	.0162	-.0050	.0511
	3	-.0010	.0267	.0143	.0260	.0143	.0260	.0105	.0475
	4	.0334	.0101	.0492	.0092	.0492	.0092	.0516	.0106
	5	.0221	.0110	.0392	.0092	.0422	.0092	.0476	.0132
	6	.0250	.0214	.0455	.0178	.0461	.0178	.0592	.0269
	7	.0366	.0541	.0563	.0466	.0571	.0465	.0700	.0541
	8	.1137	.3133	.1353	.2966	.1361	.2962	.1416	.2529
	9	.0582	.1751	.0808	.1610	.0813	.1608	.0826	.1828
71	1	.0003	.0292	-.0006	.0297	-.0006	.0297	.0086	.0409
	2	-.0006	.0806	.0003	.0844	.0003	.0844	-.0406	.2727
	3	.0627	.0335	.0645	.0323	.0645	.0323	.0727	.0423
	4	.0232	.0310	.0312	.0295	.0312	.0295	.0497	.0260
	5	-.0343	.0426	-.0211	.0404	-.0243	.0407	-.0071	.0358
	6	-.0378	.0304	-.0228	.0292	-.0244	.0295	-.0425	.0581
	8	-.2118	.3379	-.1882	.2975	-.1884	.2977	-.3484	1.6092
72	1	.0003	.0392	-.0017	.0363	-.0017	.0363	-.0042	.0588
	2	.0005	.0710	.0038	.0692	.0038	.0692	-.0316	.2209
	3	.0694	.0419	.0715	.0428	.0715	.0428	.0765	.0504
	4	.0466	.0403	.0534	.0422	.0534	.0422	.0631	.0481
	5	.0247	.0381	.0356	.0437	.0328	.0443	.0372	.0467
	6	.0811	.1905	.0957	.2096	.0938	.2109	.0603	.0815
	8	-.0550	.3290	-.0321	.3246	-.0326	.3256	-.2208	1.3616
TW-5	1	.0000	39.87	.1744	40.67	--	--	--	--
	2	-.0082	64.12	.2991	65.94	--	--	-.3419	40.49
TW-6	1	.0000	59.66	.1567	60.86	--	--	--	--
	2	.0100	78.00	.1482	81.71	--	--	-.5644	65.22
TW-7	1	.0000	20.96	.0622	21.26	--	--	--	--
	2	.0000	20.87	.0875	21.02	--	--	-.2291	9.80
TW-10	1	.0000	221.37	.2786	226.01	--	--	--	--
	2	.0000	164.43	.4778	172.11	--	--	-1.9250	51.86
TW-13	1	.0000	39.24	.2113	40.80	--	--	--	--
	2	.0000	53.96	.3400	56.99	--	--	-1.8774	32.58
TW-17	1	.0000	20.74	.1022	21.14	--	--	--	--
	2	.0000	37.13	.1727	37.74	--	--	-.4680	35.55

PREDICTED UPGROWTH, MORTALITY, AND CONVERSION; ACTUAL INGROWTH

Conversion from blackjack pine to yellow pine is applied at the end of each fourth growth period. Therefore, only simulation runs longer than four growth periods will show the effect of adding predicted conversion upon the resulting predicted diameter distribution. For simulation runs (such as the even-aged runs) under five growth periods in duration, the results are identical to those runs using actual conversion and ingrowth. As a result, the even-aged data could not be used to check the predicted conversion models.

The fifth and sixth columns of table 18 present the mean and variance of differences for the simulator using the final predicted upgrowth, mortality, and conversion models, and actual ingrowth. Results indicate that using predicted conversion does not reduce accuracy and precision by much.

ALL COMPONENTS PREDICTED

The final component, ingrowth, was developed using the uneven-aged data alone and, for this reason, is not applicable to the even-aged validation plots. The addition of the ingrowth component reduces both accuracy and precision (see the last two columns of table 18), which is particularly noticeable on the longer runs and on the managed, uneven-aged plots (plots 71 and 72).

Because an objective was to produce a simulator useful in predicting even-aged stands as well, a substitute for the ingrowth model was devised and then tested on the even-aged validation plots. This consisted of extending the use of the upgrowth and mortality models into the 1-, 2-, and 3-inch diameter classes as well. If the number of trees existing below the 1-inch diameter class and the possibility of additional regeneration are both insignificant for the stand, then ingrowth should consist of those trees from the 1-, 2-, and 3-inch classes surviving and upgrowing into the 4-inch or larger diameter classes. Therefore, if the available upgrowth and mortality equations can be extended into these classes, reasonable ingrowth predictions should result.

The values listed in the last two columns of table 18 for the Taylor Woods plots are the results of applying the proposed technique for two growth periods. On all plots, the technique both increased the mean difference per diameter class and reduced the variance of the differences. The resulting mean differences, however, are still small when compared to values for average actual number of trees per diameter class that ranged from 9.38 (for TW-7) to 77.90 (for TW-10). I concluded from these findings that the technique is an adequate substitute for an ingrowth model if it is applied to a stand with an average diameter of at least 4 inches. This is necessary because all of the Taylor Woods plots fell in this category.

Predicting Subplots and Averaging vs. Predicting Plot Averages

Of those uneven-aged whole stand simulators developed and published, all have used relatively small plots as their data base. Moser (1972) used 1/5-acre plots. Later Moser (1974) used plots ranging in size from 1 to 4 acres. Finally, Ek (1974) used 1/7-acre plots. The apparent assumption is that these small plots have the same structure and dynamics as the total stand. This assumption is probably met in truly all-aged stands; however, if clumping exists (as it does in ponderosa pine), the assumption probably cannot be met.

To test this assumption, the average plot diameter distributions were predicted and compared to the values previously obtained by averaging the predicted subplots. Table 19 presents mean differences and the variance of the differences for each plot and growth period using the method of predicting plot averages. Comparing these values to those found in the last two columns of table 18, I concluded that predicting plot averages generally reduces both accuracy and precision. The loss, however, appears not too severe for most potential uses of a whole stand simulator. The conclusion is fortunate since using the simulator for answering managerial questions would be greatly restricted if it were necessary to first simulate subplots and then average to get stand values.

Table 19.--*Mean difference and variance of differences between actual and predicted average plot number of trees per diameter class for a specified plot and growth period*

Plot	Growth period	Mean difference	Variance of difference
61	1	0.0275	0.0226
	2	.0563	.0595
	3	.0636	.0939
	4	.0474	.0125
	5	.0742	.0168
	6	.1076	.0746
	7	.1368	.1100
	8	.1745	.1451
	10	.2724	.5277
62	1	-.0224	.0449
	2	-.0464	.0647
	3	-.0607	.1314
	4	.0174	.0070
	5	.0263	.0057
	6	.0284	.0134
	7	.0300	.0119
	8	.0642	.0664
	9	-.0453	.2520
71	1	-.0137	.0780
	2	-.0847	.4184
	3	.0564	.0426
	4	.0362	.0283
	5	.0189	.0292
	6	-.0486	.0807
	8	-.4189	2.0891
72	1	-.0264	.0991
	2	-.0773	.3569
	3	.0615	.0399
	4	.0409	.0368
	5	.0469	.0196
	6	.0378	.0223
	8	-.3097	1.7529

RESULTS OF STUDY

Analysis of the validation runs demonstrates that the simulator provides reasonable results for runs at least 40 to 50 years in duration (that is, 8 to 10 growth periods). This is fortunate because cutting cycles can be from 5 to 30 years long, and the method for determining optimal diameter distribution requires good estimates of the future stand at the end of the cutting cycle. In addition, solving the optimal conversion strategy problem could require simulation runs as long as two or three cutting cycle lengths.

The final equations that make up the simulator are found in table 20. Those equations incorporating predicted basal area growth as an independent variable (the mortality and through-growth equations) have had their regression coefficients changed to adjust for the final removal of the log bias correction. A description of the control cards for the simulator can be found in appendix J.

Table 20.--*Summary of final equations for simulating uneven-aged stand dynamics in ponderosa pine/Arizona fescue habitat type*

BASAL AREA GROWTH EQUATIONS

Average Basal Area Growth

Blackjack pine

$$ln(BJBAG) = -8.51836897 + 1.16754330(ln(D)) - 4.00970143E-02(D) -3.84298771E-03(LBA_2) - 7.15483662E-03(MBA_2) - 1.58234269E-02(UBA_2) -3.26097273E-01(A_1 * ln(D)) + 8.80676713E-01(A_3) + 1.0(ln(S))$$

$$BJBAG = EXP(ln(BJBAG))$$

Yellow pine

$$ln(YPBAG) = -15.2464932 + 4.27656656(ln(D)) - 1.8316162E-01(D) -7.27361567E-05(LBA_2)^2 - 9.11165626E-04(MBA_2)^2 -2.41462106E-04(UBA_2)^2 - 1.05776062(A_1) + 1.0(ln(S))$$

$$YPBAG = EXP(ln(YPBAG))$$

Fast, Moderate, and Slow Basal Area Growths

Blackjack pine

$$\text{Fast } BJBAG = EXP(0.56557130 + 0.12995917(A_2) + ln(BJBAG))$$

$$\text{Moderate } BJBAG = EXP(0.09120012 + 0.12946423(A_2) + ln(BJBAG))$$

$$\text{Slow } BJBAG = EXP(-0.65677142 - 0.25942340(A_2) + ln(BJBAG))$$

Yellow pine

$$\text{Fast } YPBAG = EXP(0.70338067 + 0.7280557(A_3) + ln(YPBAG))$$

$$\text{Moderate } YPBAG = EXP(0.24266226 + 0.59168455(A_3) + ln(YPBAG))$$

$$\text{Slow } YPBAG = EXP(-0.94604293 - 1.31974025(A_3) + ln(YPBAG))$$

MORTALITY EQUATIONS -- Proportion of trees dying in next 5-year period

$$PM = (1.0 + EXP(-X))^{-1}$$

Blackjack pine

$$X = -5.372649 + 1.270862E-03(D^2) - 12.96478(A_1 * BJBAG)$$

Yellow pine

$$X = -3.612474 - 4.226903(YPBAG) - 22.24186(A_1 * YPBAG) + 5.597303E-04(D^2)$$

CONVERSION EQUATION -- Proportion of blackjack pine converting to yellow pine in each fourth growth period

$$PC = (1.0 + EXP(-Y))^{-1}$$

$$Y = -3.777269 + 0.5012605(D) - 6.597465E-03(D^2) - 0.0982176(S)$$

UNEVEN-AGED INGROWTH EQUATIONS

Total Ingrowth

$$TI = b_0 S(BACL_1)^{b_1}(1.0 + BACL_2)^{b_2} EXP(b_3 BA^{b_4} + b_5(A_1 * BA^{b_4}))$$

$b_0 = 0.48630258$	$b_3 = -3.97721395E-06$
$b_1 = 0.6594918$	$b_4 = 2.7667388$
$b_2 = -0.49280582$	$b_5 = -1.33299951E-06$

Through-Growth -- Proportion of total ingrowth growing into 5-inch diameter class

$$PT = (1.0 + EXP(-Z))^{-1}$$

$$Z = -2.636336 + 38.80808(BJBAG\text{-}4") - 0.03419626(A_1 * TI)$$

(continued)

Table 20 (continued)

Ingrowth into 4-inch diameter class

$ING_{4''} = TI*(1.0 - PT)$

Ingrowth into 5-inch diameter class

$ING_{5''} = TI*PT$

HEIGHT EQUATIONS

Even-aged

$$H_D = MH - 0.88015037(MH - 4.5)\left(1.0 - \frac{D}{MD}\right)^{\text{[illegible]}.35}$$

Uneven-aged

$$H_D = c_0 + c_1 \cdot S^{c_2} \cdot e^{c_3 (D+35)^{-2}}.$$

$c_0 = 4.5$

$c_1 = 13.178649$

$c_2 = 0.71631005$

$c_3 = -4221.6528$

DEFINITION OF VARIABLES

BJBAG = Predicted average 5-year basal area growth, in square feet, of blackjack pine

YPBAG = Predicted average 5-year basal area growth, in square feet, of yellow pine

BJBAG-4" = Predicted average 5-year basal area growth in the 4-inch diameter class of blackjack pine

PM = Proportion of trees dying in the next 5-year period

PC = Proportion of blackjack pine converting to yellow pine in each fourth growth period

TI = Total ingrowth for uneven-aged stands

PT = Proportion of total ingrowth growing into the 5-inch diameter class for uneven-aged stands

$ING_{4''}$ = Ingrowth into the 4-inch diameter class for uneven-aged stands

$ING_{5''}$ = Ingrowth into the 5-inch diameter class for uneven-aged stands

H_D = Average height of the D*th* diameter class

D = Diameter class size

S = Minor's (1964) site index

MBA_2 = Basal area in the given diameter class plus the two adjoining larger and smaller diameter classes

LBA_2 = Total basal area below the smallest diameter class in MBA_2

UBA_2 = Total basal area above the largest diameter class in MBA_2

BA = Total basal area

= $MBA_2 + LBA_2 + UBA_2$, for a specified diameter class

$BACL_1$ = Basal area in the 4- through 6-inch diameter classes

$BACL_2$ = Total basal area above the 6-inch diameter class

A_1 = -0.244178 + 1.244178*EXP(-(1.176471 - 0.019607*TIME)^3)

A_2 = -0.095336 + 1.095336*EXP(-(1.25 - 0.020833*TIME)^4)

A_3 = -0.000203 + 1.000203*EXP(-(1.428571 - 0.023809*TIME)^6)

TIME = Number of 5-year growth periods since last cutting

DM = Maximum diameter class size that an even-aged stand has achieved in its development

MH = Maximum height as predicted by Minor's (1964) equation

= $S - 1.4003(\sqrt{A} - 10) + 0.1559(S)(\sqrt{A} - 10)$

A = Breast height stand age, $20 \le A \le 140$

CONCLUSIONS

A simulator for predicting uneven-aged stand development was constructed and validated for the ponderosa pine/Arizona fescue habitat type of the Southwest. The structure and dynamics of the simulator make it potentially useful for answering even- and uneven-aged management questions. Some results and conclusions formulated during the construction and validation process are:

1. The distribution of number of trees within 1-inch diameter classes is uniform.

2. Diameter class basal area growth rates of even- and uneven-aged stands are logically interrelated if the structure of within stand competition is incorporated in the model.

3. No reasonable log bias correction could be found for the log of basal area growth equations when residuals are not normally distributed. In the case of nonnormality, the development of a calibrating routine is recommended as an alternative to a log bias correction.

4. Even- and uneven-aged endemic mortality rates can be logically interrelated if catastrophic mortality is treated in a fashion similar to cutting mortality.

5. The ingrowth component is the weakest link in the simulator. The development of a regeneration, seedling, and sapling component is highly desirable.

6. The use of the chi-square statistic for testing "goodness-of-fit" in validation is highly questionable.

7. The use of regression statistics for testing "goodness-of-fit" in validation did not prove to be very valuable.

8. The simulation of 2.5-acre subplot values followed by the calculation of their average to obtain mean plot values produced different results than simulating plot averages directly. Because all published uneven-aged stand simulators have used small plots, this result could prove a problem for those interested in simulating plot averages. For the simulator developed in this study, however, analysis also revealed that the difference between the two methods was not too severe.

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APPENDIX A

DATA PROBLEMS

The even- and uneven-aged plots used in this study provide an unusual modeling opportunity for western species. Unfortunately, their use also posed numerous modeling problems:

1. Narrow geographic and site range.--All of the uneven-aged plots are situated within a square mile tract, and the even-aged plots are about 1 mile away. The uneven-aged plots have a narrow range in site and, because of this, site index as an independent variable may not be significant for predictive purposes. In addition, the even-aged plots have an average site index higher than uneven-aged plots. While the differences in site index may be insignificant for the uneven-aged plots, the difference between the uneven- and even-aged plots may well be significant. All this could have contributed to the problems encountered while trying to incorporate site index in the various component models of the simulator.

Fortunately, one mitigating factor for these problems is that all the plots (both even- and uneven-aged) occur on the same habitat type, *Pinus ponderosa/Festuca arizonica* (ponderosa pine/Arizona fescue), and it is believed that the plots and their site indices are representative of the type. Therefore, the simulator developed in this study is probably applicable to that habitat type, at least in northern Arizona.

2. Changes in diameter measurement limits.--Ideally, the smaller the lower diameter limit, the better. Large, lower diameter limits ignore too much of the stand that is important in understanding and modeling stand dynamics. The changing of lower limits on the Fort Valley uneven-aged plots caused problems and raised several questions. A lower diameter limit of 7.6 inches would be necessary to utilize the full-time range of the data. This would provide six to nine 5-year growth periods of data and would incorporate cuttings at several times. It would, however, also ignore about one important third of the stand--all trees under 7.6 inches.

Conversely, a lower diameter limit of 3.6 inches would restrict the data to only the early time periods. This would provide three to four 5-year growth periods, but it would also eliminate many of the cuttings. I decided to use only the early data because: (a) a larger portion of the stand would be modeled, and (b) by including more of the total stand, the possibility of meeting objective number 2 would be increased.

Because of the lower diameter limit of 3.6 inches for the uneven-aged stands, all data on the even-aged plots below 3.6 inches were ignored. Several plots in the Taylor Woods study have many small trees and, as a result, much of their structure was eliminated. A greater portion of the number of trees in the total stand is usually located in the 0.0- to 3.5-inch class of uneven-aged stands than in most even-aged stands, with the exception of very young even-aged stands. While a lower diameter limit of 3.6 inches is an improvement over 7.6 inches, it is still far from the ideal solution of modeling the total stand.

3. Intrusion upon uneven-aged plots.--Unfortunately, a major paved highway runs through pieces of all the uneven-aged plots, and this could affect their stand dynamics. Not only would these disturbed subplots have a serious "edge effect," but the two remaining subplot pieces would not have the same competitive interaction as occurs on the undisturbed subplots. The number of subplots seriously affected by the highway are given in table 21.

Table 21.--*Number of subplots by size, plot number, and presence of highway*

Subplot size in acres	Plot 61		Plot 62		Plot 71		Plot 72	
	No highway intrusion	Highway intrusion	No highway intrusion	Highway intrusion	No highway intrusion	Highway intrusion	No highway intrusion	Highway intrusion
0.50	3							
.70	1							
.90				1				
1.00			3					
1.90							1	
2.00							4	
2.20							3	
2.30		1				2		
2.35		5				1		
2.40	1	1				1		1
2.45		1				2		
2.50	20	1	28		18		23	
2.60					1			
2.80					2			
2.90					1			
3.00					1	1		
3.05						1		
3.10					1			
	25	9	31	1	24	8	31	1
Number of subplots seriously affected by highway	6		0		5		0	

4. Variable plot sizes.--As mentioned earlier, sizes on the uneven-aged subplots range from 0.5 to 3.1 acres (table 21), and the even-aged plots from 0.75 to 1.25 acres. Curtis and Pope[2] hypothesize that "small" plots can result in erratic estimates of stand attributes, such as basal area, because of within-stand clumping. As a result, the deviation of plot estimates of growth and basal areas from their respective stand means will be correlated and, therefore, the regression of diameter class growth as a function of basal area will also reflect plot size. They conclude that this problem would not produce biased results if the regression equation were applied to inventory plots of the same size and design, and if the stands to which they are applied had similar spatial patterns. Recognizing that these conditions are seldom met, Curtis and Pope suggested that the problems would be minimized if large plots were used in developing growth equations.

Plot size is important to this study because the two data sets (even- and uneven-aged) have different plot sizes, both within and between the data sets. based on the Curtis and Pope argument, the coefficients for the "separate" models (even- and uneven-aged) may be confounded by the plot size differences, making a statistical combination more difficult. To help minimize the problem with the uneven-aged subplots, all subplots below 2 acres were eliminated. This action, coupled with the realization that the even-aged plots are fairly homogeneous (because of stand treatment) and good sized, might reduce the problem of within-stand clustering and the possibility of locating a subplot within one cluster of the stand.

[2] Curtis, Robert O., and Robert B. Pope. 1972. Some considerations in design of growth studies and associated inventories of the western National Forests. Draft of the internal report. USDA For. Serv., Pac. Northwest For. and Range Exp. Stn., Portland, Oreg.

5. No height-growth information on uneven-aged plots.--The lack of height-growth data on the uneven-aged plots did preclude the development of a common even- and uneven-aged height-growth model, but its absence was not critical to the dynamics of the basic components in the simulator. Because some uneven-aged height-diameter data were available from both the School of Forestry at Northern Arizona University and the Rocky Mountain Forest and Range Experiment Station at Flagstaff, I decided to develop separate height-diameter relationships for even- and uneven-aged stands. While height-diameter relationships are not ideal substitutes for a height-growth model because they do not incorporate the effect of stand density upon height, their use does provide a means of assessing product potential.

The School of Forestry data came from student class measurements of 1-acre square subplots made on plot 61 during 1964 and 1967. The Rocky Mountain Forest and Range Experiment Station data came from one-time measurements on selected subplots of plots 71 and 72.

APPENDIX B

DISTRIBUTION OF TREES WITHIN A DIAMETER CLASS

The uniformity of tree distribution within a diameter class is likely to be influenced by the combination of diameter class size and the number of trees in the diameter class. The fewer the number of trees in a class, the more difficult it would be to prove nonuniformity; and the number of trees in a class is, in part, determined by the diameter class size. Therefore, the smaller the diameter class size, the higher the probability that the assumption of uniformity can be met.

To test for uniformity, about one-fourth of the subplots reserved for model building were randomly selected from both the uneven- and even-aged plots. All applicable measurement periods on the selected subplots were examined, and those 1-inch diameter classes with five or more trees in them were then chosen for testing.

A chi-square "goodness-of-fit" test was selected because it is appropriate for testing the distribution of a discrete random variable (the number of trees in a class). For diameter classes with five to nine trees in them, the diameter class was divided into five equal subclasses, and for diameter classes with 10 or more trees, the number of subclasses was 10. The expected number of trees in each subclass was computed by dividing the total number of trees in the class by the appropriate number of subclasses.

The decision to use two sizes of subclasses was based on the suggestion by Snedecor and Cochran (1967) that the expected value in any subclass should be greater than or equal to one. Since the number of trees in a diameter class is usually low in the Southwest, the inclusion of the five-to-nine size group spreads the testing over more diameter class sizes. For example, of the 2,195 diameter classes with trees examined in this phase, 1,481 had less than five trees, 417 had five to nine trees, and the remaining 297 diameter classes had 10 trees or more (table 22).

Table 22.--*Results of testing within diameter class distribution for each plot by number of trees in diameter class*

Plot	Number of diameter classes				
		5 to 9 trees		10+ trees	
	1 to 4 trees	Uniform	Nonuniform	Uniform	Nonuniform
61	499	147	1	39	2
62	415	79	0	8	0
71	257	98	0	94	2
72	299	87	0	101	0
Taylor Woods	11	5	0	49	2
Total	1,481	416	1	291	6

Testing was done at the 99 percent level of significance in order not to reject the null hypothesis of a uniform distribution unless marked deviations were found. The degrees of freedom for the test were computed as the number of subclasses minus one because it was not necessary to estimate distribution parameters. Results of the tests are found in table 22.

Of the 714 diameter classes tested, only seven were not uniformly distributed. While it was earlier hypothesized that the within diameter class distribution might depend upon number of trees in the class and perhaps stand structure, the results indicate that these factors are not as important as first thought. For example, on the even-aged plots tested, 20 of the 21 diameter classes with 50 or more trees in them were uniformly distributed, indicating that the within diameter class distribution is not closely correlated with the number of trees or structure of the stand.

APPENDIX C

DEVELOPMENT OF DIAMETER GROWTH MODELS

Prior Findings

Diameter growth is influenced by several factors including productivity. A measure of productivity is site index (the height obtainable by free growing trees at a base age). Minor's (1964) site index was developed for even-aged stands in northern Arizona, and its use is recommended for the area.[3] The application of site index to uneven-aged stands is complicated by the difficulty of finding suitable free growing site trees. This complication is reduced to some extent in the uneven-aged stands because they have relatively low stocking levels; however, this problem still causes difficulties. Another problem with site in the study area is the previously mentioned narrow range of site indices computed for the plots, which may make the inclusion of a meaningful site measure difficult.

Another factor related to productivity that also influences diameter growth is rainfall. Fritts and others (1965) found that diameter growth in southwestern ponderosa pine is influenced by precipitation and temperature of the autumn, winter, and spring prior to the growth period. Pearson and Wadsworth (1941) found that gross 5-year volume increment appeared to be related to the amount of precipitation in the period. Finally, Pearson (1936) hypothesized that a fall off in 5-year diameter growth in the second period after thinning was due to lower precipitation in that period.

Total stand density can also influence diameter growth. For uneven-aged stands in the Southwest, Krauch (1940) and Pearson (1950) both reported that a reduction in stocking through cutting increased diameter growth. After evaluating growth in young, even-aged clumps of uneven-aged stands, Cooper (1961) found that average tree diameter was "...largely determined by stand density." For even-aged ponderosa pine stands in the Southwest, the effect of density upon growth has been widely reported (Pearson 1936; Hornibrook 1936; Krauch 1949b; Gaines and Kotok 1954; Cooper 1960; Larson and Minor 1968; Schubert 1971, 1974; and Myers and others 1976). In all cases, an increase in density reduced growth.

The correlation of diameter class size with growth has also been reported in the Southwest. Some have found that, following cutting in uneven-aged stands, diameter growth was not as closely correlated with diameter class size as might be expected (Pearson 1933, 1950; Pearson and Marsh 1935). Others, however, have found that diameter growth did vary markedly between classes in cut stands (Krauch 1937) and in uncut stands (Pearson 1950). For even-aged stands, both Pearson (1936) and Krauch (1949a, 1949b) reported differences in average diameter growth between diameter classes.

One feature mentioned frequently in the literature is the difference in growth rates between "yellow" pine and "blackjack" pine. In all cases, diameter growth was greater in blackjack pine than in yellow pine (Pearson 1933, 1950; Pearson and Marsh 1935; and Krauch 1934, 1937).

While not mentioned specifically in the literature for the Southwest, one additional factor that does affect diameter growth is the position of the diameter class in the stand (that is, how much competition a given stand receives from above and below it). A measure of this type can be thought of as an index of the competitive structure, and it is intimately related to stand structure. A competitive structure index might, therefore, explain differences between even-aged and uneven-aged growth rates. Competitive structure was one of the critical factors for meeting objective number two of this study.

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[3]Personal communication with Gilbert H. Schubert, principal silvilculturist with USDA For. Serv., Rocky Mountain Forest and Range Experiment Station, Flagstaff, Ariz.; retired.

General Model Form

When modeling diameter growth, one of the first decisions is the general form that the model should take. The three most common forms suggested for growth modeling are the following:

$$Y = \underline{b}' \cdot \underline{x} + \varepsilon \tag{1}$$

$$Y = f(\underline{a},\underline{x}) + \varepsilon \tag{2}$$

$$Y = f(\underline{b},\underline{x}) \cdot \varepsilon \tag{3}$$

where

Y = the dependent growth variable

$\underline{x}$ = an array of independent variables that may themselves be functions of basic growth factors

$\underline{a}$, $\underline{b}$ = arrays of model parameters to be estimated in the appropriate fashions

$f(,)$ = a nonlinear function

ε = a random error.

Model form (1) assumes that growth is a linear, additive function of transformed independent variables. The estimation of the parameters of the model is done through ordinary least squares regression techniques. Vuokila (1965) used this form when he modeled percent diameter growth of individual trees.

Model form (2) assumes that growth is a nonlinear function of the independent variables. Because the error term, ε, is additive, the appropriate technique for estimating the model parameters is nonlinear, least squares regression (Kmenta 1971).

Model form (3) differs from (2) in how the error term is introduced. In model (3), the error term is multiplicative instead of additive. This allows the error structure model to be linearized through the use of natural logarithms. For this model form to work, it is necessary that the parameters ($\underline{b}$) of the nonlinear function can also be linearized through the log transforming process. The resulting model is of the form:

$$ln\ (Y) = \underline{b}' \cdot \underline{g(x)} + \varepsilon^* \tag{4}$$

where

$\underline{g(x)}$ = an array of appropriately transformed independent variables to linearize the parameters of the the function $f(\underline{b},\underline{x})$

$\varepsilon^* = ln(\varepsilon)$

If these assumptions are met, then estimation of the parameters in model (4) can be done by ordinary least squares regression techniques. Model form (4) has been used by Lemmon and Schumacher (1962) to model individual tree periodic radial increment, and by Cole and Stage (1972) and Stage (1973) to model individual tree periodic basal area increment.

Another choice faced by the modeler of diameter growth is the form of the dependent variable. Following are some of the choices: radial growth, diameter growth, basal area growth, or any of the previous three expressed as a percentage. Cole and Stage (1972) and Stage (1973) settled upon the usage of basal area growth for two reasons:

1. Basal area growth is often nearly linear over short time periods, and this makes extrapolation of growth rates easier for growth periods different from that originally used in equation development.

2. The log of basal area growth is often found to be more normally distributed with homogeneous variance than other dependent variables.

The latter finding may also indicate that the residuals of the log of basal area growth are additive. If the assumption of normality, homogeneity of variance and additive, independent residuals can be met, then the ordinary least squares estimators, $\underline{b}$, are the maximum likelihood estimators and the UMVUE's (Uniformly Minimum-Variance Unbiased Estimator), and various procedures for testing the significance of the model and its parameters can be validly applied (Kmenta 1971; Draper and Smith 1966).

Given these various choices of model forms and dependent variables, I decided that the log of basal area growth would be used because:

1. Previous experiences with modeling diameter growth in ponderosa pine[4] indicate that a nonlinear, multiplicative model best represents the interaction of the independent variables with themselves and their effect upon diameter growth.

2. Cole and Stage (1972) and Stage (1973) have shown that basal area growth is often lognormally distributed with a multiplicative error term. This fits the requirements of model (3).

The Random Error Component

The two approaches for handling the random error component are stochastic modeling and deterministic modeling. In stochastic modeling, each component of the model with an error element is randomly assigned an error value from the proper distribution. The prediction from this component is then modified by this random disturbance, and this "randomized" prediction interacts with the other model components to produce a randomized estimate from the model (usually with an unknown error structure). This process is then repeated a number of times, each time using a new set of random errors, and the results of these numerous trials can be averaged to get an "expected" model estimate. This repetitive stochastic process is often called a "Monte Carlo" process. One disadvantage of this approach is the sometimes large number of computations needed to find the "expected" estimates.

The deterministic approach assigns each component of the model its expected value. There is no random element. Each component interacts with other components in the appropriate fashion to produce model estimates. The advantage of deterministic models is that "expected" estimates are provided directly.

Stage (1973) describes and uses a method in his prognosis model that maintains the relative simplicity of the deterministic approach while introducing a stochastic element. He calls the approach a Monte Carlo "swindle." He says, "The purpose is to produce a prognosis that overall is the result of averaging many replications of the random process without actually having to carry out the replication." The random element of this "swindle" is applied to the log of basal area growth equations in one of two ways. The particular method is dependent upon the number of trees sampled in the stand.

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[4]David W. Hann. Diameter growth equations developed for Mora County, New Mexico, and the Black Hills, South Dakota, inventories, on file with the Forest and Range Resources Evaluation Project, USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, Utah.

For large samples, Stage randomly picks a deviate from the normal distribution of the residuals about the log of basal area growth regression equation and adds it to the estimate of log of basal area growth. The underlying assumption is that, with a large sample, the effects of the individual random deviations will average out over the stand.

For small samples, each sample tree record is divided into three sample tree records. The number of trees in each record is a fixed proportion of the original number of trees represented by the old sample tree record. The proportion breakdown Stage used was 15, 60, and 25 percent, based on previous findings that an average stand had 15 percent suppressed trees and 25 percent dominant trees.[5] Each of the new sample tree records is assigned a growth rate computed by taking the average growth rate (as predicted by the log basal area growth equations) and adding a random component. For the 25 percent dominant trees, the random component is the expected residual value of the largest 25 percent of the normally distributed residuals about regression; for the 15 percent suppressed trees, it is the expected residual value of the lowest 15 percent of the normally distributed residuals, and similarly for the middle trees. As a result, the weighted log of basal area growth still sums to the average log of basal area growth, as predicted by the equation. At each simulation period, the sample tree records are split again until enough sample trees exist so that the first method can be used.

Certain aspects of the second method seem applicable to this study. Within a diameter class, individual tree growth could be quite variable. By using only average diameter growth, all trees in the class will be assigned the same growth rate and, therefore, advancement to larger diameter classes will be identical. If, however, the number of trees in a diameter class were divided into thirds (each third representing the fast, slow, and moderate growers), then a separate growth rate could be assigned each third in the same fashion as used by Stage. If the three growth rates differed enough, this would then allow the trees in a diameter class to move into a wider range of larger diameter classes. If the distribution of residuals remains constant over time, this approach should provide a more realistic estimate of the number of trees changing diameter classes. Thirds were used in this study instead of Stage's proportions because the proportion of suppressed and dominant trees for southwestern ponderosa pine is unknown.

Definition of Independent and Dependent Variables

The previously discussed factors influencing diameter growth include diameter class size, productivity, rainfall, total stand density, vigor, and position of the diameter class within the stand.

Over diameter class size (D), it is expected that basal area growth will climb to some peak value and then drop off. Where and at what magnitude the peak will occur is unknown. One function that would allow a wide range of peaking forms is the Weibull function. The Weibull function can be expressed as (Bailey and Dell 1973):

$$y = \frac{c}{b}\left(\frac{x}{b}\right)^{c-1} \cdot \mathrm{EXP}\left[-\left(\frac{x}{b}\right)^{c}\right],$$ where b and c are parameters

or

$$y = \frac{c}{b^c}\, x^{c-1} \cdot \mathrm{EXP}\left[-\frac{1}{b^c}\, x^c\right] \quad .$$

[5]Based on a presentation given by Albert R. Stage, principal mensurationist with the Intermountain Forest and Range Experiment Station, USDA Forest Service, in a spring 1976 graduate seminar at the University of Washington.

Linearizing this function gives:

$$ln(y) = ln\left(\frac{c}{b^c}\right) + (c-1)\ ln(x) - \frac{1}{b^c}\ x^c\ .$$

Generalizing this function further to provide for an even wider range of potential model forms:

$$ln(y) = b_0 + b_1 ln\ x + b_2 x^c,\quad \begin{matrix} b_{1>0} \\ b_{2<0} \end{matrix}\ .$$

Therefore, the inclusion of ln(D) and D^c in the log of basal area growth model would allow for a wider range of "peaking" functions. The values of c chosen were 1, 1.5, 2, 2.5, 3, 3.5, and 4. These values cover a wide range of forms from the Weibull function (Bailey and Dell 1973).

The measure of productivity in this study is Minor's (1964) site index. It is expected that, when site index (S) is zero, growth would be zero, and, as site increases, growth would also increase. To model this effect, the proposed independent variable is the log of site index, in other words, ln(S). This choice of independent variable is supported by the findings of Cole and Stage (1972).

Because of the findings of Fritts and others (1965) and Pearson and Wadsworth (1941), I decided to express precipitation in two fashions: average annual 5-year growth period rainfall (GRF), and average annual 5-year rainfall (ARF). Growth period rainfall is defined as the rain that fell in the interval from September through May previous to the given growing period. Based on a reasoning similar to that for site index, I decided that the log of GRF and the log of ARF were the appropriate independent variables.

There are several different expressions of total stand density, including the following: total stand basal area per acre (BA), total number of trees per acre (T), and crown competition factor (CCF) (Larson and Minor 1968). In all cases, growth should maximize when the stand density values are near zero, and growth should decline as the values increase. An appropriate form for modeling this behavior is:

$$y = b_0 e^{-b_1 x^d}$$

where

y = predicted growth

x = measure of stand density

b_0, b_1 = model parameters

d = a power on x

Linearizing this form produces the following independent variables: BA^d, T^d, and CCF^d. Three potentially suitable values of d are 2, 3, and 4 (d could take on any positive value). These particular values were chosen because any one of them would provide for a slow decline in y when x is near zero, which is also an expected effect (that is, low stand densities will not influence growth).

The vigor differences between blackjack pine and yellow pine were handled by simply modeling each of them separately.

The final class of independent variables is those indicating the position of the diameter class within the stand. For this purpose, Stage (1973) developed the independent variable "percentile in the basal area distribution" (PCT). Stage defined this variable as the basal area in all trees equal to or smaller in diameter than a given tree, divided by total stand basal area. In this study, PCT was defined in a similar fashion, substituting "diameter class" for "tree." Transforms of this variable include the following: *ln*(PCT), PCT (Stage 1973), and PCT^2.

Another "position" variable that Cole and Stage (1972) and Stage (1973) have used is diameter of the tree divided by the diameter of the tree of mean basal area (DCP). This independent variable can also adapt to the diameter class situation by dividing diameter class size by the diameter of the tree of mean basal area. Potentially, useful transforms of this variable include DCP and *ln*(DCP).

The final method of representing the position of the diameter class within the stand consists of separating total stand basal area into three components. The middle basal area component is defined as that falling in a specified diameter class (MBA_1); or within the specified diameter class plus the adjoining larger and smaller diameter classes, if they exist (MBA_2); or within the specified diameter class plus the adjoining two larger and two smaller diameter classes, if they exist (MBA_3). The lower basal area component (LBA_1, LBA_2, or LBA_3) is the total basal area existing below the smallest diameter class in MBA_i, and the upper basal area component (UBA_1, UBA_2, or UBA_3) is the total basal area existing above the largest diameter class in MBA_i.

In this fashion, three sets containing three independent variables were derived: LBA_1, MBA_1, and UBA_1; LBA_2, MBA_2, and UBA_2; and LBA_3, MBA_3, and UBA_3. These variables should behave in the same fashion as the total stand density variables, and therefore the only additional transformation used was to also square each value. This resulted in six sets, each containing three independent variables. For convenience, the definition of all independent variables and their abbreviations can be found in table 23.

Due to the desire to divide the residuals about the final models into fast, average, and slow growers, the appropriate dependent variable is the log of basal area growth for each tree. While the dependent variable is an individual tree value, it must be remembered that all independent variables are based only on diameter class information.

Development of Equations

Equations were developed for both blackjack pine and yellow pine using the least square regression program REX (Grosenbaugh 1967). REX is a powerful screening tool because it is designed to fit all combinations of independent variables following specified combinatorial rules. The screening statistic used is the relative mean square residual (RMSQR), which is defined as the mean square error about regression divided by the variance of the dependent variable. Therefore, a perfect fit results in an RMSQR value of zero, while an RMSQR value of one would indicate that the regression equation is no better at reducing squared residuals about regression than a simple mean. The advantage of the RMSQR over the more commonly used coefficient of determination (R^2) is that the former reflects the reduction in the degrees of freedom caused by adding another independent variable. Therefore, it is easier to compare models with a different number of independent variables.

Table 23.--*Definition of independent variables and abbreviations*

Abbreviation	Independent variable
D	Diameter class size (e.g., for the diameter class 8.6 to 9.5, D = 9)
S	Site index
GRF	Average annual 5-year growth period rainfall
ARF	Average annual 5-year rainfall
BA	Total stand basal area per acre
CCF	Crown competition factor
T	Total number of trees per acre
PCT	Percentile in the basal area distribution
DCP	Diameter class size divided by the diameter of mean basal area
LBA_1	Total basal area under a given diameter class
MBA_1	Total basal area in a given diameter class
UBA_1	Total basal area over a given diameter class
LBA_2	Total basal area under (a given diameter class - 1)
MBA_2	Total basal area in (a given diameter class ± 1)
UBA_2	Total basal area over (a given diameter class + 1)
LBA_3	Total basal area under (a given diameter class - 2)
MBA_3	Total basal area in (a given diameter class ± 2)
UBA_3	Total basal area over (a given diameter class + 2)
TIME	Number of 5-year periods since last cutting

The first set of screening runs was made on the separate plots (61, 62, 71, 72, and Taylor Woods separately) with the purpose of identifying those independent variables most highly correlated with the log of basal area growth, and eliminating those independent variables that proved to be poor(er) predictors. To do this, the independent variables were classified into sets, and then each set was placed in a group (table 24). A set is defined as one or more independent variables such that, when chosen by combinatorial rules, all independent variables in the set are included (or excluded when not chosen). The only sets containing more than one independent variable were those formed with the three independent variables LBA_i, MBA_i, and UBA_i, or their squares.

Table 24.--*Definition of independent variables, groups and sets for the individual plot log of basal area growth screening runs*

Variable number	Group number	Set number	Variable
1	1	1	$ln(D)$
2	2	1	D
3		2	$D^{1.5}$
4		3	D^2
5		4	$D^{2.5}$
6		5	D^3
7		6	$D^{3.5}$
8		7	D^4
9	3	1	$ln(ARF)$
10		2	$ln(GRF)$
11	4	1	BA^2
12		2	BA^3
13		3	BA^4
14		4	CCF^2
15		5	CCF^3
16		6	CCF^4
17		7	T^2
18		8	T^3
19		9	T^4
20	5	1	$ln(PCT)$
21		2	$PCT_{\frac{1}{2}}$
22		3	$PCT^{\frac{1}{2}}$
23		4	$ln(DCP)$
24		5	DCP
25		6	LBA_1
26			MBA_1
27			UBA_1
28		7	$(LBA_1)^2$
29			$(MBA_1)^2$
30			$(UBA_1)^2$
31		8	LBA_2
32			MBA_2
33			UBA_2
34		9	$(LBA_2)^2$
35			$(MBA_2)^2$
36			$(UBA_2)^2$
37		10	LBA_3
38			MBA_3
39			UBA_3
40		11	$(LBA_3)^2$
41			$(MBA_3)^2$
42			$(UBA_3)^2$

A group contains one or more sets. The five groups in these screening runs were established on the following criteria: group 1 contained the single variable, $ln(D)$; group 2 consisted of the seven powers on D; group 3 was the two rainfall variables; group 4 was the total stand density variables; and group 5 was the position in diameter class sets. Site index was not included in these runs because the difference between the site index values on each half plot was probably too small to prove significant in modeling. Program REX was then set up to tabulate the RMSQR values of all regression equations formed by choosing, at most, one set from each group.

From these screening runs, 12 blackjack and yellow pine equations were selected and the regression coefficients determined. These coefficients were next examined for reasonableness of behavior and for consistency of performance between plots. In addition, the model with the lowest RMSQR was picked for each plot and the residuals of these models were checked for normality. To do this, skewness and kurtosis statistics were computed using formulas found in Kendall and Stuart (1977). From this examination, I concluded that the residuals are not normally distributed, that they are highly skewed, and are leptokurtic (Kendall and Stuart 1977). One consequence of the lack of normality is that all of the usual significance tests that could be made on regression results are not applicable.

Another problem that made testing difficult or impossible was the lack of data range and overlap between the different stand conditions. The site index of the even-aged plots was larger than the uneven-aged plots; the stand structures were obviously different; and the even-aged plots covered only a narrow range of diameter classes. For the uneven-aged stands, the managed and virgin plots differed in stand structure due to cutting. If legitimate tests were available, a test indicating significant differences in the models of each stand condition might be due to the data being truly incompatible, or merely that the the data are complementary. Because of these problems, I concluded that testing to see if individual data sets could be pooled was not practical.

The main question was whether the even-aged and uneven-aged data sets could be combined for blackjack pine. To answer this without testing, it was decided to develop two blackjack pine equations: one with the even-aged data, and one without. Both of these could then be evaluated as to their predictive capability.

A second set of screening runs was then made with the plots combined as such: for both blackjack pine and yellow pine, the virgin, uneven-aged plots (61 and 62) were combined; and the cut, uneven-aged plots were combined (71 and 72). In addition, another set of blackjack pine data was created by combining the Taylor Woods even-aged data with the cut, uneven-aged data. From the findings of the first screening, the independent variables T, PCT, and DCP (and their transformations) were eliminated. Site index (S) was added as another group.

The results of this second set of screening runs were examined, and 11 blackjack pine and 4 yellow pine equations were selected and the regression coefficients determined. As before, these coefficients were also checked for reasonableness of behavior and for consistency of performance between data sets. For blackjack pine, those independent variables that both behaved reasonably and minimized RMSQR were $ln(D)$, D, $ln(S)$, $ln(GRF)$, LBA_2, MBA_2, and UBA_2. For yellow pine, the independent variables included $ln(D)$, D, $ln(S)$, $ln(GRF)$, $(LBA_2)^2$, $(MBA_2)^2$, and $(UBA_2)^2$. The independent variable of $ln(S)$ was inconsistent for blackjack pine and unreasonable for yellow pine. It was included, however, because of the desire to develop a simulator applicable for the range of site indices found in the habitat type. The problems with site index persisted throughout the analysis and finally necessitated special handling, which will be discussed later.

The differences in the size of coefficients between the uncut and cut data sets were then modeled as a function of time-since-last-cutting (TIME), expressed as the number of 5-year intervals since the last cutting. The uncut virgin data sets were given a value of 60 (or 300 years). I hypothesized that the transition from "cut" regression coefficients to "uncut" regression coefficients would be along a smooth path, and therefore three sigmoidal curve forms were developed using MATCHACURVE techniques (Jensen and Homeyer 1970). All three curves are zero when TIME is zero and one when TIME equals 60. The differences in the three curves are their forms (fig. 9), and they were chosen more or less subjectively. Because the TIME data fell at both extremes, I felt that a more objective approach was not possible. The three equations are:

$$A_1 = -0.244178 + 1.244178*EXP(-(1.176471 - 0.019607*TIME)^2)$$

$$A_2 = -0.095336 + 1.095336*EXP(-(1.25 - 0.020833*TIME)^4)$$

$$A_3 = -0.000203 + 1.000203*EXP(-(1.428571 - 0.023809*TIME)^6)$$

where

TIME = time-since-last-cutting expressed in number of 5-year growth periods.

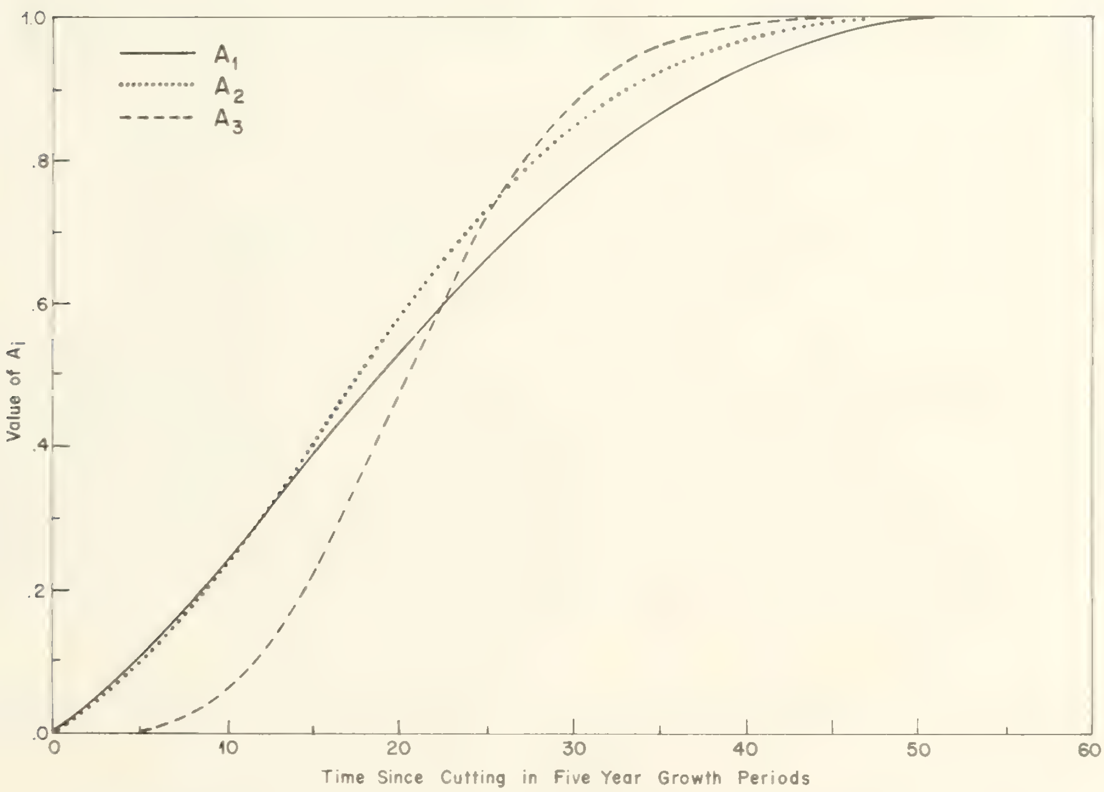

Figure 9.--Time-since-last-cutting sigmoidal curves.

Using these time-since-last-cutting variables, a third set of screening runs was made to combine all data sets for each "vigor" class. Again, blackjack pine had two data sets; one with Taylor Woods data and one without. Table 25 lists the independent variables and how they were grouped. The runs were set up so that the basic independent variables from the previous step were forced into all equations. The screening then picked, at most, one independent variable from each of the remaining groups. These groups were formed by multiplying the three sigmoidal curves (A_1, A_2, A_3) by all the independent variables except $ln(S)$ and $ln(GRF)$. For those independent variables in which a time-since-last-cutting variable is also picked, the effect is to provide for a change to the regression coefficient of that independent variable as time-since-last-cutting changes.

Table 25.--*Definition of independent variables, groups and sets for combined plot, log of basal area growth screening runs*

Variable number	Group number	Set number	Variable[1]
1		1	$ln(D)$
2			D
3			LBA_2 or $(LBA_2)^2$
4			MBA_2 or $(MBA_2)^2$
5			UBA_2 or $(UBA_2)^2$
6			$ln(S)$
7			$ln(GRF)$
8	1	1	$A_1 ln(D)$
9		2	$A_2 ln(D)$
10		3	$A_3 ln(D)$
11	2	1	$A_1 D$
12		2	$A_2 D$
13		3	$A_3 D$
14	3	1	$A_1 LBA_2$ or $A_1 (LBA_2)^2$
15		2	$A_2 LBA_2$ or $A_2 (LBA_2)^2$
16		3	$A_3 LBA_2$ or $A_3 (LBA_2)^2$
17	4	1	$A_1 MBA_2$ or $A_1 (MBA_2)^2$
18		2	$A_2 MBA_2$ or $A_2 (MBA_2)^2$
19		3	$A_3 MBA_2$ or $A_3 (MBA_2)^2$
20	5	1	$A_1 UBA_2$ or $A_1 (UBA_2)^2$
21		2	$A_2 UBA_2$ or $A_2 (UBA_2)^2$
22		3	$A_3 UBA_2$ or $A_3 (UBA_2)^2$
23	6	1	A_1
24		2	A_2
25		3	A_3

[1]In those cases where two variables are given, the first is for blackjack pine and second is for yellow pine.

The most promising equations from these runs were selected and the regression coefficients determined. Analysis of these coefficients showed that the coefficients on $ln(S)$ were not reasonable (that is, they were negative) for yellow pine and for blackjack pine with the uneven-aged data set alone (plots 61, 62, 71, and 72), and that the coefficient on $ln(GRF)$ was not reasonable for blackjack pine with the uneven- and even-aged data sets combined. These runs also indicated that a high degree of multicollinearity existed.

Multicollinearity and Ridge Regression

Multicollinearity results when one independent variable is highly correlated with another, or with a linear combination of other independent variables. While the model is still an unbiased estimator, the effect of multicollinearity is to produce highly imprecise estimates of each regression coefficient (Kmenta 1971). This high degree of impreciseness is due to the multicollinearity between independent variables causing the correlation matrix to approach singularity (Farrer and Glauber 1967). One possible effect is that some of the regression coefficients may be of the wrong sign from what was expected (Hoerl and Kennard 1970a, 1970b).

One method used for reducing multicollinearity effects is ridge regression. This sacrifices unbiasness to obtain estimates that, when compared to their unbiased least squares counterparts, are often interpretable and have a smaller mean square error. In terms of solving for the standardized regression coefficients,[6] the method consists of adding a small constant value, K, to the diagonal elements of the correlation matrix and then solving in the usual manner for the regression coefficients. When K is zero, the ordinary least squares regression estimates result. While K can be any positive value, it usually lies between zero and one. Also, the larger the value of K, the larger the bias. Details of ridge regression can be found in Hoerl and Kennard (1970a, 1970b), Brown and Beattie (1975), Marquardt and Snee (1975), or Hocking (1976).

Numerous methods have been proposed for determining which value of K is "best"; Hocking (1976) summarizes these methods. One technique common to many of these methods is the development of a ridge trace, which was first proposed by Hoerl and Kennard (1970a, 1970b). The ridge trace is usually a plot of the resulting standardized regression coefficients over a range of their respective K values, usually from zero to one. Using the ridge trace, Hoerl and Kennard (1970a) suggested four items to consider when deciding upon a value of K:

1. At a certain value of K, the system will stabilize and have the general characteristics of an orthogonal system.

2. Coefficients will not have unreasonable absolute values with respect to the factors for which they represent rates of changes.

3. Coefficients with apparently incorrect signs at K = 0 will have changed to have the proper sign.

4. The residual sum of squares will not have been inflated to an unreasonable value. It will not be large relative to the minimum residual sum of squares or large relative to what would be a reasonable variance for the process generating the data.

Because of the high degree of multicollinearity in the data and of the unreasonable signs on some of the independent variables, I decided to try ridge regression to determine if it could improve the estimates of the regression coefficients. I wrote a program to compute regular and standardized regression coefficients for K values ranging from zero to 0.95 in intervals of 0.05. These values could then be plotted to produce the ridge trace.

---

[6] Standarized regression in coefficients, b_i^*, and normal regression coefficients, b_i, are related in the following manner:

$$b_i = \frac{s_y}{s_{x_i}} \cdot b_i^*$$

where

s_y = the variance of the dependent variable.

s_{x_i} = the variance of the ith independent variable.

The results for yellow pine indicated that most of the independent variables were stable. Two exceptions are the independent variables D and *ln*(D). Their behavior was common to all data sets and to all models, and was expected because of the high correlation between the two. Disappointingly, ridge regression did not change the sign on *ln*(S). For blackjack pine fitted to the uneven-aged data alone, the results were similar. The sign of *ln*(S) did not change, and the four independent variables involving D all showed instability. This last was the most notable feature of the ridge trace for blackjack pine fitted to both the even- and uneven-aged data sets.

Because of the desire to have a model that was applicable over the range of site indices found in the habitat type, the results of the ridge regression runs were unacceptable. I decided, therefore, to force a reasonable value of *ln*(S) on the models, which was done by fitting a new dependent variable of *ln*(BAG/S). This has the same effect as forcing the independent *ln*(S) onto the log of basal area growth model with a regression coefficient of one. The choice of one as the regression coefficient was made because, fixing other factors, growth increased in direct proportion to an increase in site index (W. H. Meyer 1938; Larson and Minor 1968; Ek 1974; and Myers and others 1976).

Given this new dependent variable, a final set of screening runs was made utilizing the same strategy as that used for the third set. Promising models were chosen from these runs and additional runs were made to determine their regression coefficients. Ridge regression was then used to examine the stability of the various independent variables.

As with the ridge trace for the log of basal area growth, the independent variables showing the greatest amount of instability are those involving D. For both blackjack pine equations, a moderate amount of instability was also exhibited by those variables involving the same basic variables (for example A_3*UBA_2 and UBA_2).

The instability between D and *ln*(D) was expected because of their high correlation, but this was acceptable because the two independent variables are necessary to provide the desired "peaking" effect over diameter class size. For those independent variables involving time-since-last-cutting, however, I decided to eliminate those that were highly unstable and therefore did not greatly improve the least squares regression equations. This resulted in no independent variables being removed for yellow pine, but it did result in three independent variables being removed from both blackjack pine equations. The ridge traces for the finished equations are found in figures 10, 11, and 12.

For two reasons, the independent variable *ln*(GRF) was also eliminated. First, the fact that the variable was negative for the blackjack pine equation with the even-aged data was cause for concern. The even-aged data were collected 40 years later than the uneven-aged data. The sign change caused by the incorporation of the even-aged data might therefore signal that the influence of rainfall was not uniform over time. Also, the data needed to compute *ln*(GRF) are not readily available. The data source used in this study came from the Fort Valley Experimental Forest, which has a weather station.

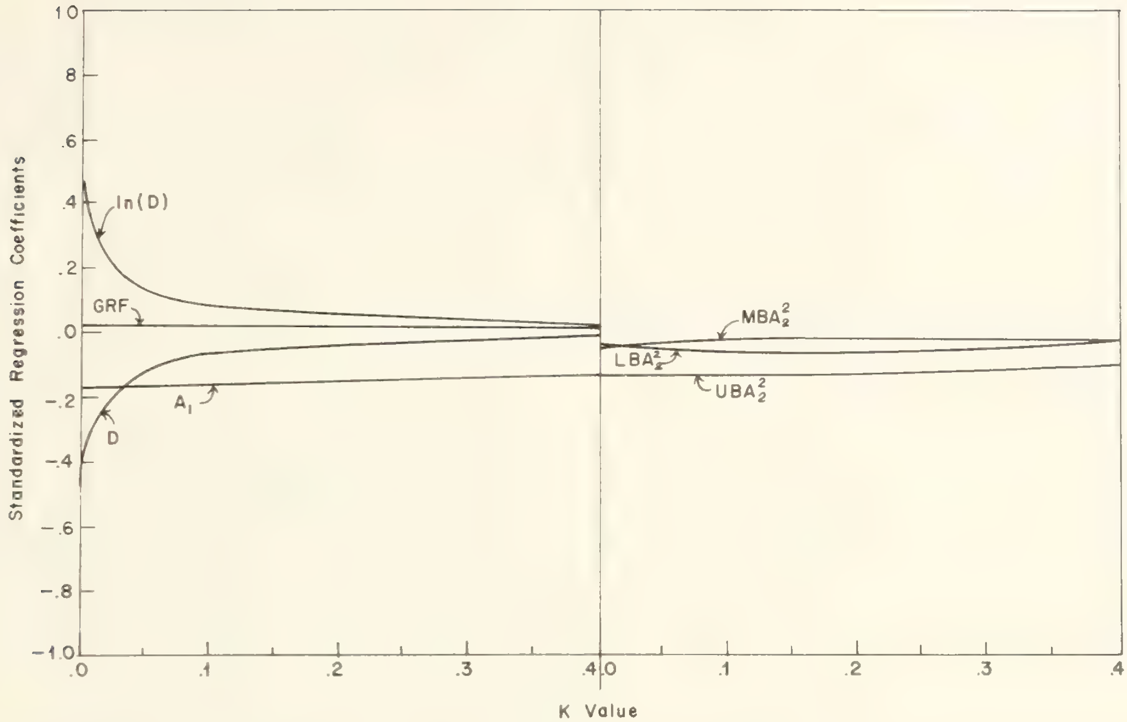

Figure 10.--Final ridge trace (with growth period rainfall) for yellow pine using the dependent variable of *ln*(basal area growth/site index).

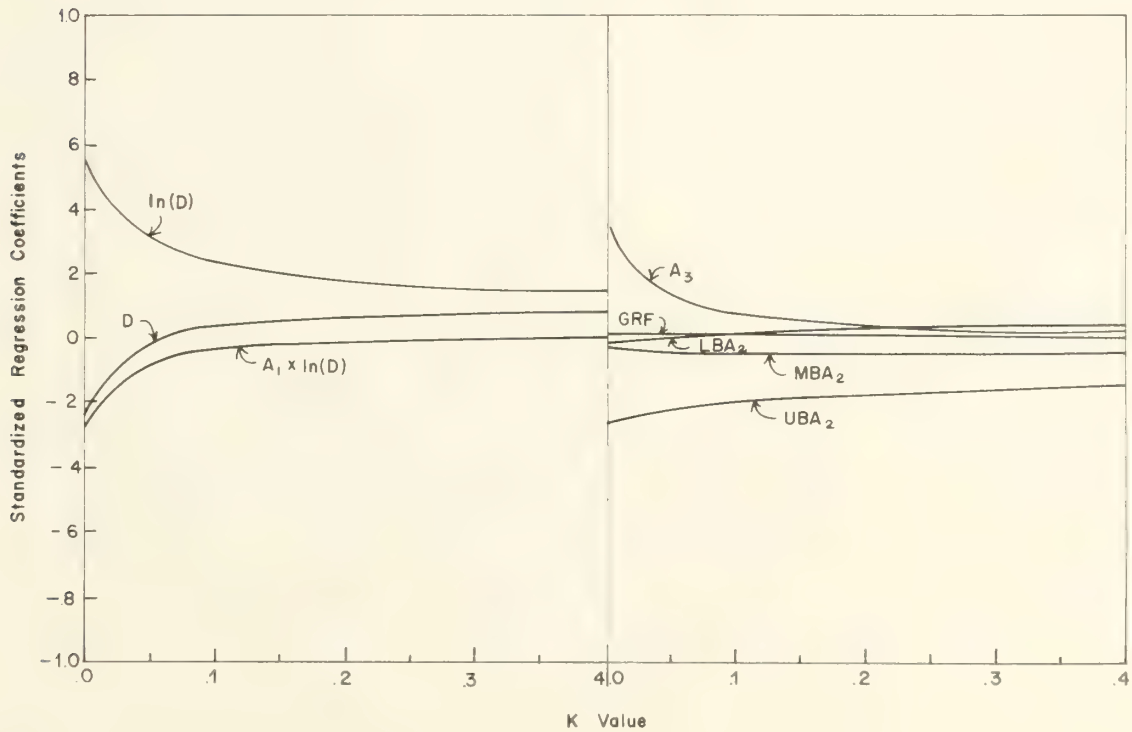

Figure 11.--Final ridge trace (with growth period rainfall) for blackjack pine fitted to the uneven-aged data alone with the dependent variable of *ln*(basal area growth/site index).

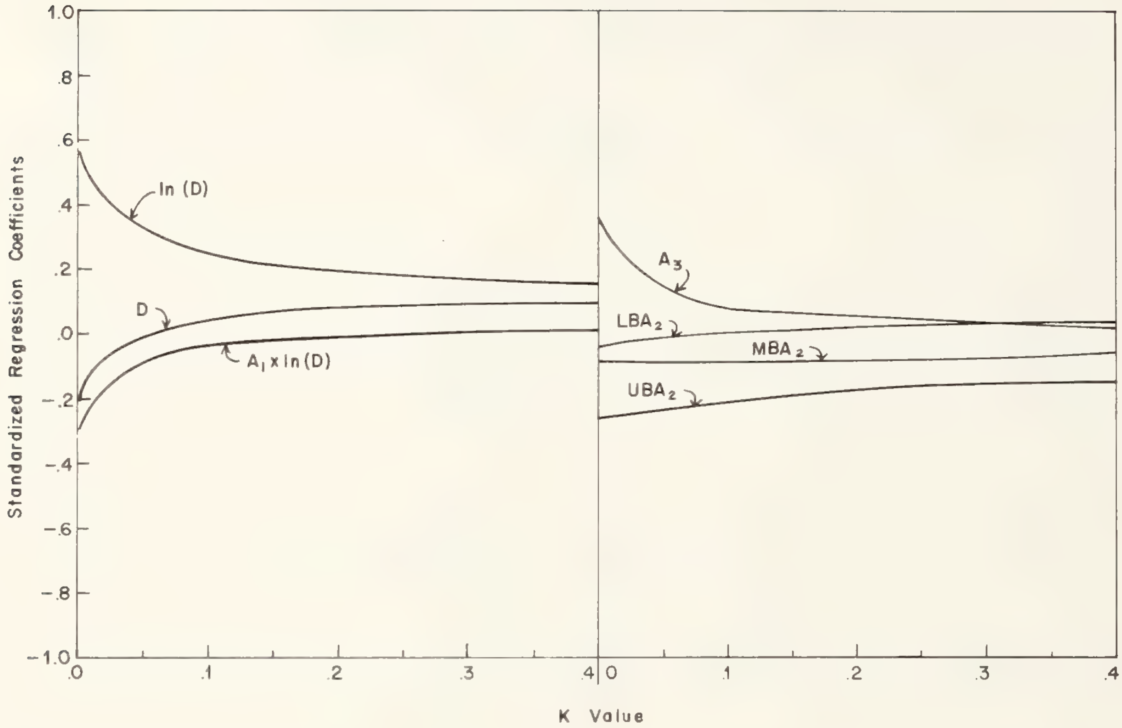

Figure 12.--Final ridge trace for blackjack pine fitted to the even- and uneven-aged data with the dependent variable of *ln*(basal area growth/site index).

APPENDIX D

CORRECTION FOR LOG BIAS

If the assumption of normality of residuals cannot be met, then correction of a log model for bias can be a problem for which no easy solution is apparent. The following, however, is a proposed correction factor based more on intuition than mathematical rigor:

If S is such that $ln S \sim N(\mu, \sigma^2)$ then it can be shown that:

$$E[S] = EXP[\mu + \tfrac{1}{2}\sigma^2] \quad \text{(Mood and others 1974)}$$

Therefore, an estimator of $\overline{S}$ is:

$$\overline{S} = EXP[\overline{ln(S)} + \tfrac{1}{2} \cdot MSE]$$

This correction for lognormal bias, $\tfrac{1}{2} \cdot MSE$, is the same as that proposed by Oldham (1965) and Baskerville (1972) as an approximate correction to the log regression model. Now, if S is not lognormally distributed, it would still seem reasonable to assume that a constant, K, still exists such that:

$$\overline{S} = EXP[\overline{ln(S)} + K] \ .$$

Rearranging,

$$\overline{S} = EXP[\overline{ln(S)}]\ EXP[K]$$

$$EXP[K] = \frac{\overline{S}}{EXP[\overline{ln(S)}]}$$

$$K = ln(\overline{S}) - \overline{ln(S)}.$$

The regression equation:

$$ln(Y) = b_0 + b_1 x_1 + \cdots + b_n x_n$$

can also be expressed as:

$$ln(Y) - \overline{ln(Y)} = b_1(x_1 - \overline{x}_1) + b_2(x_2 - \overline{x}_2) + \cdots + b_n(x_n - \overline{x}_n)$$

or,

$$ln(Y) = \overline{ln(Y)} + b_1(x_1 - x_1) + b_2(x_2 - x_2) + \cdots + b_n(x_n \quad x_n) \ .$$

If $K = ln(\overline{Y}) - \overline{ln(Y)}$ were added to this model as an approximate correction for log-bias in the nonnormal case, then the effect would be:

$$
\begin{aligned}
ln(Y) &= K + \overline{ln(Y)} + b_1(x_1 - \overline{x}_1) + b_2(x_2 - \overline{x}_2) + \cdots + b_n(x_n - \overline{x}_n) \\
&= ln(\overline{Y}) - \overline{ln(Y)} + \overline{ln(Y)} + b_1(x_1 - \overline{x}_1) + \cdots + b_n(x_n - \overline{x}_n) \\
&= ln(\overline{Y}) + b_1(x_1 - \overline{x}_1) + b_2(x_2 - \overline{x}_2) + \cdots + b_n(x_n - \overline{x}_n) .
\end{aligned}
$$

In other words, the correction would force the model through the log of mean Y instead of the mean of log Y. Therefore, Y will pass through $\overline{Y}$ when $x_1 = \overline{x}_1$, $x_2 = \overline{x}_2$, $\cdots$, $x_n = \overline{x}_n$, which appears to be the adjustment desired. Whether this is a true "mean-unbiased" estimator is impossible to prove without first knowing the true distribution of $ln(Y)$.

The correction for log bias is applied by adding it to the intercept term of the log of basal area growth model. The log bias correction factors for the three equations developed in this study are found in table 11. For comparative purposes, the appropriate correction for normally distributed residuals (that is, ½•MSE) is also found in table 26. Notice that if normality had been assumed, the correction would have been considerably larger than the one not assuming normality.

Table 26.--*Proposed correction factor and log-normal correction factor for log bias of the three log of basal area growth equations*

Equation type	Proposed factor [*ln*(mean Y)] - [mean *ln*(Y)]	Lognormal factor ½•MSE
Blackjack pine with even-aged data	0.300534069	0.48453144
Blackjack pine without even-aged data	.324720185	.56464436
Yellow pine	.873936593	2.76746302

APPENDIX E

DEVELOPMENT OF MORTALITY MODELS

Prior Findings

For uneven-aged stands, noncatastrophic mortality has been found to be correlated to diameter size, severity of cutting, and whether the trees were blackjack or yellow pines. Krauch (1926) reported that percent mortality increased as diameter increased. When percent mortality is plotted over diameter, the curve is U-shaped with the minimum near 20 inches (Pearson 1939, 1950; Pearson and Wadsworth 1941; Wadsworth and Pearson 1943; and Myers and Martin 1963).

For severity of cutting, Krauch (1926) discovered that "the percentage of mortality increases in inverse ratio to the degree of cutting." This discovery was supported by Lexen (1935). Wadsworth and Pearson (1943) found that percent mortality of blackjack or yellow pine alone was higher in a cutover stand than in a virgin stand, but that this situation reversed itself when the two were combined. They attributed this reversal to peculiarities of percentages. Pearson (1950) also found that a virgin stand had a higher rate of mortality than a moderately cut stand, but that severe cutting increased mortality because of windthrow and lightning losses in the few large trees remaining. Higher mortality rates in yellow pine than in blackjack pine have been reported by Pearson and Wadsworth (1941) and by Wadsworth and Pearson (1943).

Mortality should also be related to productivity, total stand density, and time-since-last-cutting. For even-aged stands, Myers and others (1976) found that site and total stand basal area were significant in predicting total stand mortality, and there is no reason to believe this effect is not also significant in uneven-aged stands.

While it seems intuitive that the longer the time-since-last-cutting, the higher the mortality rate, both Lexen (1935) and Pearson (1939, 1950) have reported that they could find no consistent relationship between mortality and time-since-last-cutting. Their findings could have been clouded by other effects or by the fact that the length of time-since-last-cutting they examined (20 years or less) was not long enough for the effect to manifest itself.

Definition of Variables and Transformations

A mortality rate can be expressed in several ways. Ek (1974) and Adams and Ek (1974) expressed it directly as the number of trees dying in a diameter class during the growth period. Lee (1971) expressed mortality as a percentage of the total stand dying per year. Finally, Hamilton (1974), Hamilton and Edwards (1976), and Monserud (1976) all expressed mortality as the probability (or proportion) of a tree dying in a growth period.

The form of the equations has also differed. Ek (1974) used nonlinear least squares techniques, while Lee (1971) and Adams and Ek (1974) used ordinary, linear least squares techniques. Hamilton (1974), Hamilton and Edwards (1976), and Monserud (1976) all used the logistic function fitted using weighted, nonlinear least squares techniques.

Expressing mortality as a percentage or proportion has an advantage over expressing it as the number of trees dying. If properly modeled, a proportion (or percentage) is bounded by 0 and 1 (or 0 and 100). Multiplying this rate by the number of trees in the class will always give a value less than or equal to the number in the class. As a result, predicted number of survivor trees is never negative; a result possible under the other approach.

A problem can arise with using ordinary, linear least squares regression for predicting the percent or proportion of trees dying. Linear models can cause the predicted values to exceed zero or one, especially if the prediction falls outside the range of values in the developmental data set (Hamilton 1974; Hamilton and Edwards 1976). However, the nonlinear logistic function,

$$y = (1.0 + e^{-(b_0 + b_1x_1 + b_2x_2 + \cdots + b_nx_n)})^{-1}$$

limits the value of y to between zero and one. Also, when the dependent variable is dichotomous (which mortality is), the logistic function appears to have improved statistical properties over ordinary, linear least squares regression models (Hamilton 1974; Hamilton and Edwards 1976).

Hamilton (1974) presents a computer program, RISK, for determining the parameters of the logistic function using weighted, nonlinear least squares regression techniques. In using RISK, the dependent variable is assigned a value of zero if the tree survived to the end of the growth period, and a value of one if it did not. By using only diameter class or stand attributes as independent variables, the result is the probability of a tree dying under the specified diameter class and stand conditions.

The reported U-shaped curve of percent mortality over diameter was for all ponderosa pine. By separating ponderosa pine into the two vigor classes of blackjack pine and yellow pine, the trends will differ from the U-shaped curve. For blackjack pine, it is possible that percent mortality will decline as diameter increases; while for yellow pine, the opposite is expected. Because blackjack pine usually represents the small, vigorous trees and yellow pine the large, overmature trees, it is easy to see why the resulting combined graph could be U-shaped.

Any transform on diameter class size (D) that causes the transformed value to increase as D increases would allow the desired effect in the logistic function. The two independent variables chosen, D and D^2, both meet this criterion and are also simple and interpretable (D is straightforward, and D^2 is analogous to diameter class basal area). As with the basal area growth equations, vigor was incorporated into the equations by developing separate mortality equations for blackjack pine and for yellow pine.

Two measures of productivity were tried: site index for the one-half plot (S), and average 5-year growth period rainfall (GRF). Mortality will decline as productivity increases; therefore, the independent variables chosen were the untransformed values S and GRF. A positive sign on the regression coefficients of these independent variables would provide the desired or expected effect.

The choice of the independent variables to represent stand density was based, in part, on the results of the growth analysis. The sets of LBA_2, MBA_2, UBA_2, and of $LBA_2{}^2$, $MBA_2{}^2$, $UBA_2{}^2$, proved best in the growth analysis at representing both the effect of stand density and of diameter class position. Also chosen for analysis in this phase were the independent variables of total stand basal area (BA) and percentile in the basal area distribution (PCT).

For time-since-last-cutting, two of the three transforms used in the growth analysis (A_1 and A_3) were also tried in this analysis. Expressing severity of cutting was a problem. On the plots with cutting, the recording of data did not start until after the cut. Thus, information concerning the amount of trees removed was not available as an index of severity of cutting. While the amount of the residual stand might provide some insight into the severity of cutting, it is not the best measure. Because stand density reflects the amount of the residual stand, it was felt that additional measures were neither justified nor necessary.

With the inherent difficulty of predicting mortality, I decided to include an independent variable that incorporated all of the above independent variables into one. The obvious choice for independent variable is predicted basal area growth. The probability of mortality would decrease as basal area growth increased. Predicted basal area growth was used instead of actual basal area growth because all independent variables must be observable when the model is actually applied in the stand simulator (Monserud 1976). The use of actual basal area growth in equation development and then predicted basal area growth in application of the equation would introduce a random variable as an independent variable, which violates one of the assumptions of least squares regression. To clarify this last point, consider the following example:

Suppose you wanted to obtain an estimator of Y as a function of $\hat{X}$ (the predicted value of X) because once the estimator of Y is determined, the only value available to activate the estimator is $\hat{X}$. However, the observed values (Y, X) do exist prior to the development of the estimator. Now if Y and $\hat{X}$ are related by

$$Y = \beta_0 + \beta_1\hat{X} + \varepsilon_1 \qquad (1)$$

and if

$$X = \hat{X} + \varepsilon_2 \text{ or} \qquad (2)$$

$$X - \varepsilon_2 = \hat{X}, \qquad (3)$$

then, if X (the observed value) is used instead of $\hat{X}$ (the predicted value) to develop equation (1), the effect is to model:

$$Y = \beta_0 + \beta_1(X-\varepsilon_2) + \varepsilon_1 \qquad \text{(3) into (1)}$$

$$= \beta_0 + \beta_1 X-\beta_1\varepsilon_2 + \varepsilon_1$$

and Kmenta (1971) has shown that the effect of this is to make the estimators of β_0 and β_1 inconsistent.

The three predicted basal area growths used were: blackjack pine with the even-aged data (BJBAG1), blackjack pine without the even-aged data (BJBAG2), and yellow pine (YPBAG).

"Screening" Independent Variables

Program RISK helped determine which of the independent variables best predicted mortality. To do this, a run was created to fit 14 equations for the three blackjack pine data sets and 11 equations for the two yellow pine data sets. The three blackjack pine data sets were (1) virgin uneven-aged data, (2) managed uneven-aged data, and (3) managed even-aged data; and the two yellow pine data sets were (1) virgin uneven-aged data, and (2) managed uneven-aged data. These preliminary data sets were chosen to both reduce the expense of running RISK on large data sets, and to allow an examination and subsequent modeling of the trends in model parameters across the various data sets as functions of stand structure and time-since-last-cutting, if necessary.

For both blackjack pine and yellow pine, the results of this "screening" run indicated that the only independent variables with predictive strength were D, D^2, and the basal area growth equations. Because the variable D^2 proved to be better than D, it was used in the remainder of the analysis.

The next step was modeling the change in model parameters across time-since-last-cutting and stand structure. To model the effect of stand structure, two new variables were introduced: $RMBA_i$ and $RUBA_i$, which were computed by dividing MBA_i (or UBA_i) by total stand basal area. These variables are based on the hypothesis that an even-aged stand would exhibit more basal area in the middle basal area class (or less of its total basal area in the upper basal area class) than would an uneven-aged stand.

Utilizing the time-since-last-cutting and two of the "stand structure" variables ($RUBA_3$ and $RMBA_2$) as independent variables themselves or as multipliers with the other variables, a second set of "screening" runs was made on three data sets: blackjack pine with the even-aged data, blackjack pine without the even-aged data, and yellow pine. From these runs, the two most promising models from each data set were selected for more detailed analysis. The independent variables for these models are found in table 27. Both blackjack pine growth equations were used in each of the two blackjack pine data sets because no choice has been made as to which blackjack pine growth equations will be final.

Table 27.--*Independent variables in mortality models selected from each data set for final analysis*

Data set	Model number	Independent variables
Blackjack pine with even-aged data	1	BJBAG1, $RMBA_2$
	2	BJBAG1, $RUBA_3$
	3	BJBAG2, $RMBA_2$
	4	BJBAG2, $RUBA_3$
Blackjack pine without even-aged data	1	BJBAG1, A_2*BJBAG1
	2	A_1*BJBAG1, D^2
	3	BJBAG2, A_1*BJBAG2
	4	A_1*BJBAG2, D^2
Yellow pine	1	YPBAG, A_1*YPBAG
	2	YPBAG, A_1*YPBAG, D^2

Up to this point, the selection of the "best" model has been made by jointly considering four factors: (1) reasonableness of signs on the independent variables; (2) overall significance of the model as indicated by an "F" statistic in RISK; (3) maximization of "*t*" values on each parameter, as computed by RISK; and (4) minimization of a chi-square "goodness-of-fit" value also computed by RISK. Of the three measures of fit computed in RISK (R, *t*, and chi-square), Hamilton (1974) preferred the chi-square. Unfortunately, the chi-square tables in RISK used predicted event classes too wide for the mortality data used in this study. As a result, the observations fell in very few of the classes. To correct for this, a program was written to compute the chi-square statistic over a reasonable range of predicted mortality classes and also over diameter classes.

Using this program, chi-square values were determined for the models selected for final analysis. Analysis of the results indicated that both yellow pine equations appear to perform well. Because model 2 (the model with YPBAG, A_1*YPBAG and D^2) has slightly better chi-square values, it was chosen as the best yellow pine mortality model. The goodness-of-fit test across diameter classes is found in table 10.

The results for the blackjack pine equations developed using just the uneven-aged data were also satisfactory. The chi-square values for all the tests were not significant at the 99 percent level (the testing level). The signs on the coefficients of models 1 and 3 were, however, of opposite values. This unexpected result caused the following: for stands recently cut, the higher the basal area growth rate, the higher the mortality rate; while for virgin stands, the opposite was true. The coefficients on models 2 and 4 were reasonable, and so they were picked as the best models for uneven-aged blackjack pine. Tables 11 and 12 provide the goodness-of-fit tests across diameter classes for the final models.

The chi-square values indicated poor fits for the equations developed using both the even- and uneven-aged data, and this was disappointing. So a new group of runs was tried. The independent variables used in these runs are found in table 28. Of these, the two best sets of equations were models 19, 20, and models 21, 22. While the models are an improvement, they are still significantly different at the 99 percent testing level used. The goodness-of-fit tests across diameter classes for the best models (models 21 and 22) are found in tables 13 and 14.

Table 28.--*Independent variables used in final set of runs for even- and uneven-aged blackjack pine mortality*

Model number	Independent variables
5	BJBAG1, D^2, A_1, LBA_2, MBA_2, UBA_2
6	BJBAG2, D^2, A_1, LBA_2, MBA_2, UBA_2
7	BJBAG1, D^2, A_1, MBA_2
8	BJBAG2, D^2, A_1, MBA_2
9	BJBAG1, A_1, MBA_2
10	BJBAG2, A_1, MBA_2
11	BJBAG1, D^2, A_1, $A_1{}^2$, MBA_2
12	BJBAG2, D^2, A_1, $A_1{}^2$, MBA_2
13	BJBAG1, D^2, TIME, $TIME^2$, MBA_2
14	BJBAG2, D^2, TIME, $TIME^2$, MBA_2
15	BJBAG1, A_1, $A_1{}^2$, MBA_2
16	BJBAG2, A_1, $A_1{}^2$, MBA_2
17	BJBAG1, TIME, $TIME^2$, MBA_2
18	BJBAG2, TIME, $TIME^2$, MBA_2
19	BJBAG1, D^2, TIME, $TIME^{-1}$, MBA_2
20	BJBAG2, D^2, TIME, $TIME^{-1}$, MBA_2
21	BJBAG1, D^2, TIME, $TIME^{-2}$, MBA_2
22	BJBAG2, D^2, TIME, $TIME^{-2}$, MBA_2

Examining causes for mortality provides insight into the problem of modeling the combined uneven- and even-aged mortality data. For the uneven-aged data, the principal causes of mortality were lightning, wind, bark beetles, and mistletoe (Pearson 1939; Wadsworth and Pearson 1943; Myers and Martin 1963). The amount would indicate that these are noncatastrophic forms of mortality. The main causes for mortality in the even-aged plots were snowbreak and rust (Schubert 1971, 1974). Of the 160 trees that died on those subplots used in equation development, 86 percent died because of snowbreak in the first growth period after cutting, and 12 percent died of rust in the second period.

A problem exists as to whether snowbreak should be considered noncatastrophic or catastrophic mortality. Accelerated mortality after cutting is not unique to the Taylor Woods study. Also, windthrow after cutting has been widely reported (Alexander 1973 and 1975; Reukema 1970; Reukema and Pienaar 1973; and Williamson and Price 1971) while both Boldt (1970) and Williamson (1976) reported serious problems with snowbreak. Barrett (1965) also noted that "extensive precautions were taken to protect the stand from damage by porcupines and snowbend, the two greatest threats to maintaining the designated tree densities." The problem of snowbreak and bending has been described as being common to the Southwest, particularly among young dense stands such as the Taylor Woods subplots (Pearson 1950 and Schubert 1974).

Should, therefore, a mortality agent described as common be considered as a natural result of thinning? The answer is subjective and cannot be presented satisfactorily in this study. To examine how the inclusion of the Taylor Woods mortality data affects the prediction of uneven-aged mortality, the chi-square goodness-of-fit tests were made to the even- and uneven-aged mortality equations using just the uneven-aged data. The resulting chi-square statistics indicated that the inclusion of the even-aged data has deteriorated the predictive capability for the uneven-aged plots. Whether this result indicates that snowbreak is non-normal (that is, catastrophic) mortality is not clear. A comparison of the goodness-of-fit tests across diameter class (tables 29 and 30) to those for the uneven-aged equations (tables 11 and 12) indicates that the greatest differences occur in the 4- through 6-inch diameter classes, and these are the classes where most of the Taylor Woods data occurred. Despite these problems, I decided to also test these two equations in the validation phase along with the uneven-aged blackjack and yellow pine equations. Perhaps the equations developed with the even-aged data will prove to be better predictors over the long run.

Table 29.--*Chi-square test across diameter classes for best even- and uneven-aged blackjack pine mortality equation (using basal area growth equation for even- and uneven-aged blackjack pine) compared to uneven-aged data only*

Diameter class	Number of trees in class	Actual mortality	Predicted mortality	Chi-square value
4	4,416	11	45.57	26.2
5	4,066	18	35.90	8.9
6	3,322	18	24.72	1.8
7	2,629	17	17.62	0
8	2,021	8	11.44	1.0
9	1,551	7	7.84	0.1
10	1,203	5	5.58	.1
11	1,004	6	4.22	.7
12	893	3	3.54	.1
13	737	2	2.75	.2
14	686	1	2.53	.9
15	666	6	2.61	4.4
16	685	4	2.63	.7
17	649	2	2.52	.1
18	610	1	2.38	.8
19	473	0	1.96	2.0
20 - 21	908	6	4.07	.9
22 - 23	622	9	3.06	11.5
24 - 25	353	1	2.13	.6
26+	261	0	2.50	2.5
				Chi-square = 63.7

Table 30.--*Chi-square test across diameter classes for best even- and uneven-aged blackjack pine mortality equation (using basal area growth equation for uneven-aged blackjack pine only) compared to uneven-aged data only*

Diameter class	Number of trees in class	Actual mortality	Predicted mortality	Chi-square value
4	4,416	11	44.93	25.6
5	4,066	18	35.80	8.9
6	3,322	18	24.81	1.9
7	2.629	17	17.74	0
8	2,021	8	11.55	1.1
9	1,551	7	7.92	0.1
10	1,203	5	5.65	.1
11	1,004	6	4.29	.7
12	893	3	3.62	.1
13	737	2	2.82	.2
14	686	1	2.61	1.0
15	666	6	2.71	4.0
16	685	4	2.75	.6
17	649	2	2.63	.2
18	610	1	2.49	.9
19	473	0	2.05	2.1
20 - 21	908	6	4.15	.8
22 - 23	622	9	3.03	11.8
24 - 25	353	1	2.04	.5
26+	261	0	2.34	2.3
				Chi-square = 62.8

APPENDIX F

DEVELOPMENT OF A CONVERSION MODEL

I hypothesized that the following independent variables might prove useful for predicting the conversion rate: diameter class size (D), site index (S), total number of trees 7.6 inches and larger (TT8), total basal area in trees 7.6 inches and larger (BA8), total number of trees in the diameter class (TTDC), and total basal area in the diameter class (BADC). Except for site index, I expected that as each of these independent variables increased, the conversion rate would increase. The opposite would hold true for site index.

The expected relationships between the proposed independent variables and the conversion rate were based upon how changes in these independent variables would affect growth. Growth and vigor are intimately related, and it was expected that a decrease in growth rate would cause an increase in the conversion rate. Therefore, any of the proposed independent variables that would reduce the growth rate as they increased in value would also be expected to increase the conversion rate.

Based on the same reasoning as discussed in the mortality rate section, I decided to express the conversion rate as a proportion. To model this, program RISK was again used. The dichotomous dependent variable was defined as 1.0 if the tree converted to a yellow pine, and a value of 0.0 if it did not.

Twenty-two regression runs were then made using the data reserved for modeling. These runs were selected to cover most, if not all, reasonable combinations of the independent variables. The runs allowed decisionmaking at various points during the analysis on what combinations of independent variables to examine next. Chi-square values across diameter classes and across predicted conversion classes for the three best models were then examined. In all cases, the predicted values were significantly different from the actual values at the 99 percent testing level. An analysis of the chi-square values for the best model indicated the primary area of misfit was in the 12-inch diameter class. A review of the basic data also revealed that all of the conversions in the 11-, 12-, and 13-inch diameter classes and two of the three conversions in the 14-inch diameter class occurred on subplot 13 of plot 61.

I then decided to circumvent the problem of lack of fit by strengthening the data base. This was done by adding to the conversion data base those subplots originally eliminated because of a highway running through them. Using this expanded data set, the 25 new runs were made using the same procedures as described for the first set of runs. Again, the chi-square values for the three best models indicated a significant difference between predicted and actual values. As before, the lack of fit was due to the high number of conversions on subplot 13 in the 11-, 12-, 13-, and 14-inch diameter classes.

I eliminated subplot 13 from the data base as being atypical of the conversion process, then made the 10 runs listed in table 31. This time, the chi-square values of the three best models (5, 9, and 10) indicated no significant difference at the 99 percent testing level between predicted and actual conversion values across predicted conversion classes. Across diameter classes, only model 9 did not differ significantly from the actual data (table 16) and was therefore chosen as the final model fitted to this data set.

To test model 9 further, the model was checked against validation data. Across predicted conversion classes, the chi-square value for the validation data was 140.9 with 6 d.f., while across diameter classes it was 178.8 with 7 d.f. In both cases, the difference between predicted and actual conversion was significant at the 99 percent testing level. An examination of the validation data showed that, like subplot 13 of the original data, subplot 16 exhibited a large number of conversions in the 12-inch diameter class. Why this peculiarity exists in this diameter class on these two subplots is unknown, but I eliminated subplot 16 from the validation data also. The resulting chi-square fit for this modified validation data set was 3.4 with 7 d.f. across diameter classes (table 32) and 3.4 with 6 d.f. across predicted conversion classes. Both of these chi-square statistics were insignificant, indicating a good fit to the modified validation data set.

Table 31.--*Listing of conversion models tested using expanded data set minus subplot 13*

Model number	Independent variables
1	D, S, TT8, BA8
2	D, S, TT8, BADC
3	D, S, TTDC, BA8
4	D, S, TTDC, BADC
5	D, TT8
6	D, TTDC
7	D^2, TT8
8	D^2, TTDC
9	D, D^2, S
10	D, S

Table 32.--*Validation chi-square test across diameter classes for final conversion model*

Diameter class	Number of trees in class	Actual conversion	Predicted conversion	Chi-square value
4-15	747	2.00	1.23	0.5
16-19	84	1.00	1.34	.1
20-21	42	1.00	1.36	.1
22	24	1.00	1.05	0
23	26	1.00	1.38	.1
24-25	23	1.00	1.57	.2
26-28	13	3.00	1.29	2.3
29+	11	2.00	1.65	.1
			Chi-square statistic =	3.4

APPENDIX G

DEVELOPMENT OF RECRUITMENT MODELS

Prior Findings

Natural regeneration is the process in which an adequate supply of viable seed is produced, dispersed over the area, germinated, and then survives to produce a full stocking of established seedlings. The amount of seed a tree produces depends upon tree size, vigor, level of competitive stress, and presence of cone-damaging agents (Larson and Schubert 1970; Schubert 1974) as well as environmental factors. Schubert (1974) concluded that good cone crop years occur at intervals of 3 to 4 years. The amount of viable seed is positively correlated with size of cone crop (Larson and Schubert 1970; Schubert 1974).

Seed is disseminated in the fall and generation occurs the following summer (Schubert 1974). Germination follows adequate rainfall and, if this takes place too late in the season or fails to take place at all, mortality rates can be extremely high (Pearson 1950; Schubert 1974). Second-year seedlings are also highly susceptible to drought. Therefore, successful regeneration requires adequate rainfall in two consecutive years (Meagher 1950). Seedling survival is also heavily influenced by the amounts of competing vegetation (Pearson 1942, 1950; Schubert 1974).

The ingrowth rate, therefore, should be influenced by the potential of the stand to produce cones, which is positively correlated with tree size (Larson and Schubert 1970), by the level of competition, and by the time since the last good seedling year. Another factor that might influence the ingrowth rate is the structure of the competition. For example, an even-aged stand and an uneven-aged stand could have the same potential for producing cones and the same overall level of competition, but because the diameter structures of the two stands are different, the ingrowth rates might differ. (The same total basal area spread over many diameter classes in the uneven-aged stand may produce a different level of competition on a given diameter class from the same total basal area concentrated in a narrow range of diameter classes in an even-aged stand.)

The only whole-stand ingrowth models found in the literature were the two tried by Moser (1972, 1974) and one reported by Ek (1974). Moser's differential equations were of the form:

$$d(\text{Ingrowth})/dt = b_1 e^{b_2 \text{QMD}} \qquad (1) \quad \text{(Moser 1972)}$$

and

$$d(\text{Ingrowth})/dt = b_0 + b_1^{\text{BA}} \qquad (2) \quad \text{(Moser 1974)}$$

where

QMD = quadratic mean stand diameter

BA = total stand basal area.

Integrating these functions over the time interval t_0 to $(t_0 + 5)$ provides a means for converting these differential forms to equations that represent ingrowth in any 5-year growth period. The integrated equations are:

$$\text{Ingrowth} = t \cdot b_1 e^{b_2 \text{QMD}} \Big|_{t_0}^{t_0 + 5}$$

$$= (t_0 + 5) b_1 e^{b_2 \text{QMD}} - t_0 b_1 e^{b_2 \text{QMD}}$$

$$= 5 b_1 e^{b_2 \text{QMD}}$$

$$= b_1^* e^{b_2 \text{QMD}} \tag{3}$$

and

$$\text{Ingrowth} = b_0 t + b_1 \text{BA} \Big|_{t_0}^{t_0 + 5}$$

$$= 5b_0 + 5b_1 \text{BA}$$

$$= b_0^* + b_1^* \text{BA} \tag{4}$$

Ek's (1974) model predicts the total number of ingrowth trees directly by the equation:

$$\text{Ingrowth} = b_1 \text{T}^{b_2} e^{(-b_3 \text{BA}^{b_4} \cdot \text{T}^{-1})} \tag{5}$$

where

T = total number of trees

BA = total stand basal area.

Moser used nonlinear regression to fit model (1) and linear regression to fit model (2). Ek also used nonlinear regression to fit model (5). Both are assuming, therefore, that the error term is additive, but neither provided evidence to support this assumption.

Total Ingrowth Model Development

The previously discussed literature review for ponderosa pine in the Southwest indicated that ingrowth might be influenced by the potential of the stand to produce cones, the level and structure of competition, and the time since the last good seedling year.

From the data presented by Larson and Schubert (1970), the following equation was developed to predict the number of cones a tree in a given diameter class (greater than or equal to 12 inches) would produce in a year:

$$\text{Number of cones per tree per year} = 0.94993974(\text{D}-12) + 0.61427282(\text{D}-12)^2$$

$$\text{D} \geq 12$$

where

D = diameter class size.

Using this equation, the predicted number of cones (C) for the stand was computed and used as an independent variable. The number of cones is expected to be positively correlated with ingrowth.

To represent level and structure of competition, six diameter classes were created (4 to 6 inches, 7 to 9 inches, 10 to 15 inches, 16 to 21 inches, 22 to 27 inches, 28+ inches), and the number of trees and basal area in each was determined. These classes can then be combined in numerous ways to produce various measures of the level and structure of competition.

Three additional diameter classes were also created by determining the 1-inch diameter class in which the mean stand diameter exists and then adding to it the surrounding one, two, or three 1-inch diameter classes. The number of trees and the basal area in each of these three "mean stand diameter classes" were then determined.

Because of insufficient data, the time since the last good seedling year was not incorporated into this study.

Other independent variables thought to be important for predicting ingrowth were site index, mean stand diameter, and quadratic mean stand diameter. Moser's (1972) work suggested the latter two independent variables. The definitions of all variables used to predict total ingrowth are found in table 33.

In the first phase of the regression analysis, a set of screening runs was made on the virgin, uneven-aged data and on the managed, uneven-aged data using program REX. The variables were divided into sets and groups as defined in table 34, and the combinations selected were made by picking, at most, one set from each group. This resulted in 6,334 regressions being examined for each data set.

From these screening runs, seven of the most promising models were selected, their regression coefficients were determined, and then examined for reasonableness of behavior between data sets. A preliminary analysis was also made concerning normality of residuals. The skewness and kurtosis statistics indicated that the residuals were not badly "nonnormal," and therefore "t" tests could be used to roughly check the significance of the regression coefficients.

Based on the reasonableness and significance criterion, the six original basal area classes (BC_1, BC_2, BC_3, BC_4, BC_5, BC_6) were collapsed, first to the three classes (BC_1; $BC_2 + BC_3$; $BC_4 + BC_5 + BC_6$) and ultimately to two classes (BC_1; $BC_2 + BC_3 + BC_4 + BC_5 + BC_6$). The latter two classes were redefined as $BACL_1$ and $BACL_2$.

The next step was to combine the virgin and managed data sets. The two best models from the previous set of runs were chosen, and two new screening runs were made. These runs "forced" the basic models upon the combined data sets and then allowed adjustment for time-since-last-cutting by screening all combinations of the products of the independent variables with the three time-since-last-cutting variables (A_1, A_2, and A_3). (The definitions of groups, sets, and variables are found in table 35.)

An examination of the runs disclosed that the signs of the coefficients on both *ln*(site index) and *ln*(4-inch diameter class basal area growth) were negative. This was not considered reasonable. The desire to have site index as a component of the total ingrowth model dictated that a reasonable function of site index be "forced" upon the model by fitting *ln*(Total Ingrowth/Site Index) as the dependent variable. This approach was the same as used earlier in developing the basal area growth equations. Another screening run was performed using the same strategy as for model 1 of table 35; except the *ln*(site index) independent variable was eliminated. From this run, I concluded that the basal area classes could be further collapsed to the final two classes $BACL_1$ and $BACL_2$. A final screening run was then made to provide the following log model:

Table 33.--*Definition of variables used to model ingrowth*

Variable	Definition
BJBAG1-4"	Predicted basal area growth of the 4-inch diameter class using the even- and uneven-aged blackjack pine equation
BJBAG2-4"	Predicted basal area growth of the 4-inch diameter class using the uneven-aged blackjack pine equation
S	Minor's (1964) site index
C	Predicted number of cones per acre per year
MD	Mean stand diameter
QMD	Quadratic mean stand diameter
TIME	Time-since-last-cutting
T	Total number of trees per acre
BA	Total basal area per acre
BC_1	Basal area in the 4-inch through 6-inch diameter classes
BC_2	Basal area in the 7-inch through 9-inch diameter classes
BC_3	Basal area in the 10-inch through 15-inch diameter classes
BC_4	Basal area in the 16-inch through 21-inch diameter classes
BC_5	Basal area in the 22-inch through 27-inch diameter classes
BC_6	Basal area in the 28+-inch diameter classes
TC_1	Total number of trees in the 4-inch through 6-inch diameter classes
TC_2	Total number of trees in the 7-inch through 9-inch diameter classes
TC_3	Total number of trees in the 10-inch through 15-inch diameter classes
TC_4	Total number of trees in the 16-inch through 21-inch diameter classes
TC_5	Total number of trees in the 22-inch through 27-inch diameter classes
TC_6	Total number of trees in the 28+-inch diameter classes
BMD_1	Basal area in the diameter class of mean stand diameter plus the adjacent diameter classes
BMD_2	Basal area in the diameter class of mean stand diameter plus the adjoining four diameter classes
BMD_3	Basal area in the diameter class of mean stand diameter plus the adjoining six diameter classes
TMD_1	Number of trees in the diameter class of mean stand diameter plus the adjacent diameter classes
TMD_2	Number of trees in the diameter class of mean stand diameter plus the adjoining four diameter classes
TMD_3	Number of trees in the diameter class of mean stand diameter plus the adjoining six diameter classes

Table 34.--*Specification of groups, sets, and variables for first set of total ingrowth screening runs*

Variable number	Group number	Set number	Variable
1	1	1	*ln*(BJBAG1-4") or *ln*(BJBAG2-4")
2	2	1	$ln(C)$
3	3	1	$ln(S)$
4	4	1	$ln(MD)$
5		2	$ln(QMD)$
6	5	1	$ln(T)$
7		2	$ln(BA)$
8		3	$ln(BC_1)$
9			$ln(BC_2)$
10			$ln(BC_3)$
11			$ln(BC_4)$
12			$ln(BC_5)$
13			$ln(BC_6)$
14		4	$ln(BMD_1)$
15		5	$ln(BMD_2)$
16		6	$ln(BMD_3)$
17		7	$ln(TC_1)$
18			$ln(TC_2)$
19			$ln(TC_3)$
20			$ln(TC_4)$
21			$ln(TC_5)$
22			$ln(TC_6)$
23		8	$ln(TMD_1)$
24		9	$ln(TMD_2)$
25		10	$ln(TMD_3)$
26	6	1	T
27		2	BA
28		3	$BA^{1.5}$
29		4	BC_1
30			BC_2
31			BC_3
32			BC_4
33			BC_5
34			BC_6
35		5	BMD_1
36		6	BMD_2
37		7	BMD_3
38		8	TC_1
39			TC_2
40			TC_3
41			TC_4
42			TC_5
43			TC_6
44		9	TMD_1
45		10	TMD_2
46		11	TMD_3

Table 35.--*Specification of groups, set, and variables for two models in second set of total ingrowth screening runs*

Variable number	Group number	Set number	Variable	Variable number	Group number	Set number	Variable
		Model 1				Model 2	
1			$ln(S)$	1			ln(BJBAG1-4") or ln(BJBAG2-4")
2			$ln(BC_1)$	2			$ln(BC_1)$
3			$ln(BC_2 + BC_3)$	3			$ln(BC_2 + BC_3)$
4			$ln(BC_4 + BC_5 + BC_6)$	4			$ln(BC_4 + BC_5 + BC_6)$
5			$BA^{1.5}$	5			$BA^{1.5}$
6	1	1	$A_1 ln(BC_1)$	6	1	1	$A_1 ln(BC_1)$
7		2	$A_2 ln(BC_1)$	7		2	$A_2 ln(BC_1)$
8		3	$A_3 ln(BC_1)$	8		3	$A_3 ln(BC_1)$
9	2	1	$A_1 ln(BC_2 + BC_3)$	9	2	1	$A_1 ln(BC_2 + BC_3)$
10		2	$A_2 ln(BC_2 + BC_3)$	10		2	$A_2 ln(BC_2 + BC_3)$
11		3	$A_3 ln(BC_2 + BC_3)$	11		3	$A_3 ln(BC_2 + BC_3)$
12	3	1	$A_1 ln(BC_4 + BC_5 + BC_6)$	12	3	1	$A_1 ln(BC_4 + BC_5 + BC_6)$
13		2	$A_2 ln(BC_4 + BC_5 + BC_6)$	13		2	$A_2 ln(BC_4 + BC_5 + BC_6)$
14		3	$A_3 ln(BC_4 + BC_5 + BC_6)$	14		3	$A_3 ln(BC_4 + BC_5 + BC_6)$
15	4	1	$A_1 BA^{1.5}$	15	4	1	$A_1 BA^{1.5}$
16		2	$A_2 BA^{1.5}$	16		2	$A_2 BA^{1.5}$
17		3	$A_3 BA^{1.5}$	17		3	$A_3 BA^{1.5}$
18	5	1	A_1	18	5	1	$A_1 ln$(BJBAG1-4") etc.
19		2	A_2	19		2	$A_2 ln$(BJBAG1-4") etc.
20		3	A_3	20		3	$A_3 ln$(BJBAG1-4") etc.
				21	6	1	A_1
				22		2	A_2
				23		3	A_3

$$ln(\text{Total Ingrowth}) = b_0 + b_1 ln(\text{BACL}_1) + b_2 ln(\text{BACL}_2) + b_3 \text{BA}^{1.5} + b_4 \text{A}_1 \text{BA}^{1.5} + ln(\text{S}) \quad (6)$$

where

b_0 = -7.75817566

b_1 = 0.706329932

b_2 = 1.97156496

b_3 = -8.04453668E-03

b_4 = -2.69619979E-03

BACL_1 = basal area in the 4- through 6-inch diameter classes

BACL_2 = basal area in the 7+-inch diameter classes

BA = total basal area

A_1 = time-since-last-cutting transform

S = Minor's site index.

The skewness statistic for this model was 1.10468, and the kurtosis statistic was 3.55524. These statistics indicate a moderate amount of "nonnormality." Examination shows the residuals were divided into two groups. One group represented the zero valued ingrowth observations (which has been set to a very small positive number when fitting the log models).

The strange behavior of the residuals did not appear reasonable. Therefore, the anti-log of model 6 was fitted to the total ingrowth data through linear, least squares regression through the origin, to produce the following model:

$$\text{Total ingrowth} = a_0 \text{S}(\text{BACL}_1)^{b_1}(\text{BACL}_2)^{b_2}\, e^{(b_0 + b_3 \text{BA}^{1.5} + b_4 \text{A}_1 \text{BA}^{1.5})} \quad (7)$$

where

a_0 = 1.41650289

$b_0, b_1, \ldots, b_4$ = regression coefficients from model (6).

The skewness statistic for this model was 0.818989, and the kurtosis statistic was 2.44519. These values indicate that the residuals about model (7) are more normally distributed than those about model (6). Also, the residuals were in one group about the model, and they increased, indicating a necessity to weight the model.

To determine the proper weighting scheme, the residuals were divided into seven predicted total ingrowth classes, and the variance for each class was then computed. Using these data, the following model was then developed:

$$\text{Variance} = 8.42590476 + 3.11147074(\text{YHAT})^{0.804660585} \quad (8)$$

where

YHAT = predicted ingrowth from model (7).

The reciprocal of predicted variance then provides the weights necessary in weighted regression.

Because of the behavior of the residuals, it was concluded that a weighted nonlinear regression model was the most appropriate model for total ingrowth. Using the model form and initial parameter values of model (7) and the weights derived from model (8), a weighted, nonlinear regression run was made to provide the following model:

$$\text{Total ingrowth} = c_0 S(\text{BACL}_1)^{c_1} (\text{BACL}_2)^{c_2} e^{(c_3 \text{BA}^{c_4} + c_5 A_1 \text{BA}^{c_4})} \qquad (9)$$

where

c_0 = 0.42994711

c_1 = 0.66118156

c_2 = -0.46026739

c_3 = -4.8091408E-06

c_4 = 2.7316853

c_5 = -1.6118274E-06.

This model behaves well except when BACL_2 approaches zero, which causes the model to "blow up." To control this, a value of 1.0 was added to BACL_2, and the final weighted, nonlinear model was fitted.

Through-Growth Model Development

Program RISK was used to develop the through-growth model. It was hypothesized that through-growth would most likely be correlated with predicted basal area growth in the 4-inch diameter class (BJBAG1-4" and BJBAG2-4") and with predicted total ingrowth (TINGRO). Using these two independent variables and the time-since-cutting variable, A_1, RISK runs 1 through 10 listed in table 36 were made. Analysis of these runs indicated that the two best independent variables for predicting through-growth were BJBAG1-4" (or BJBAG2-4") and A_1 *TINGRO. Using these independent variables, runs 11 and 12 were made, and the chi-square "goodness-of-fit" statistics across predicted basal area growth classes and across predicted through-growth classes were computed. Across predicted basal area growth classes, the tests were insignificant at the 99 percent testing level (tables 37 and 38). Across predicted through-growth classes, the model with BJBAG1-4" was also insignificant; however, the model with BJBAG2-4" was significant. This latter test result was very surprising because, up to this point, the two blackjack pine basal area growth equations behaved consistently in relation to each other. Comparison of the mean squared errors, "*t*"-values, regression coefficients, and mean predicted basal area growths all indicate that the two independent variables, BJBAG1-4" and BJBAG2-4", and the two models do not differ greatly. This conclusion is further substantiated by the similarity of the results for chi-square tests across predicted basal area growth classes. It was decided, therefore, to ignore this chi-square test and accept model 12 as being the final model for BJBAG2-4".

Table 36.--*Listing of through-growth models tested*

Model number	Independent variables
1	BJBAG1-4", TINGRO, A_1*BJBAG1-4", A_1*TINGRO, A_1
2	BJBAG2-4", TINGRO, A_1*BJBAG2-4", A_1*TINGRO, A_1
3	BJBAG1-4", TINGRO, A_1
4	BJBAG2-4", TINGRO, A_1
5	BJBAG1-4", TINGRO, A_1*BJBAG1-4"
6	BJBAG2-4", TINGRO, A_1*BJBAG2-4"
7	BJBAG1-4", TINGRO, A_1*TINGRO
8	BJBAG2-4", TINGRO, A_1*TINGRO
9	BJBAG1-4", TINGRO
10	BJBAG2-4", TINGRO
11	BJBAG1-4", A_1*TINGRO
12	BJBAG2-4", A_1*TINGRO

Table 38.--*Chi-square test across predicted basal area growth classes for final through-growth model involving BJBAG1-4"*

Predicted basal area growth class	Total number of in growth trees in class	Actual through-growth	Predicted through-growth	Chi-square value
0.000 - 0.035	80	18	11.92	3.1
.035 - .040	553	82	94.57	1.7
.040 - .045	715	134	139.15	.2
.045 - .050	493	112	105.78	.4
.050 - .055	614	133	141.42	.5
.055 - .060	431	120	110.95	.7
.060 - .065	267	67	75.97	1.1
.065 - .070	142	34	44.75	2.6
.070+	50	20	21.73	.1
			Chi-square statistic =	10.4

Table 38.--*Chi-square test across predicted basal area growth classes for final through-growth model involving BJBAG2-4"*

Predicted basal area growth class	Total number of in growth trees in class	Actual through-growth	Predicted through-growth	Chi-square value
0.000 - 0.035	80	18	12.11	2.9
.035 - .040	553	82	95.19	1.8
.040 - .045	715	134	139.54	.2
.045 - .050	493	112	105.75	.4
.050 - .055	614	133	141.12	.5
.055 - .060	431	120	110.62	.8
.060 - .065	267	67	75.63	1.0
.065 - .070	142	34	44.61	2.5
.070+	50	20	21.86	.2
			Chi-square statistic =	10.3

APPENDIX H

DEVELOPMENT OF HEIGHT EQUATIONS

Even-aged Height Equation

An examination of the Taylor Woods height data showed that average diameter class height increased asymptotically to the height predicted by Minor's (1964) height equation, as the diameter class size approached the maximum for the stand. Functionally, this behavior can be expressed:

$$H_D = MH - b_1(MH-4.5)(1.0 - \frac{D}{DM})^n.$$

With this function, average height of the diameter class (H_D) approaches the height predicted by Minor's (1964) equation (MH) as diameter class size (D) approaches the maximum diameter class size that the stand has achieved in its development (DM). Conversely, H_D approaches $4.5 \cdot b_1$ as D approaches zero. How quickly H approaches H_D depends upon the value of n; the larger its value the quicker H will approach H_D as D increases.

The following model was used to determine the appropriate values of b_1 and n:

$$1.0 - (\frac{H_D-4.5}{MH-4.5} = b_1 \cdot (1.0 - \frac{D}{MD})^n .$$

By varying n between 1.0 and 2.5 at increments of 0.05, 30 different independent variables were formed. An all-combination screening run using least squares regression through the origin was then used to first select the best value of n and then to compute the regression parameter(b_1). The resulting RMSQR for the final model was 0.1419.

Uneven-Aged Height Equation

A number of model forms have been proposed for characterizing height as a function of diameter, including:

$$H = 4.5 + a_1 \cdot e^{a_2(D+K)^{-m}} \qquad (1)$$

$$H = 4.5 + b_1 \cdot D^{b_2} \cdot e^{b_3(lnD)^2} \qquad (2)$$

The parameters K and m of model (1) have either been fixed at values of zero and one respectively (Curtis 1967; Embry and Gottfried 1971; Burkhart and Strub 1974), or they have been included as regression parameters (Monserud 1975). Values of the parameters have been determined either through the logarithm transformation and linear regression process for models (1) and (2) (Curtis 1967; Embry and Gottfried 1971; Burkhart and Strub 1974), or they have been determined through nonlinear regression for model (1) (Monserud 1975).

To incorporate the expected effect of site index upon these height-diameter models, the following models were designed:

$$H = 4.5 + c_1 \cdot S^{c_2} \cdot e^{c_3(D+K)^{-m}} \qquad (3)$$

$$H = 4.5 + d_1 \cdot S^{d_2} \cdot e^{d_3 \cdot S \cdot (D+K)^{-m}} \qquad (4)$$

$$H = 4.5 + e_1 \cdot S^{e_2} \cdot D^{e_3} \cdot e^{e_4(lnD)^2} \qquad (5)$$

Models (3) and (4) are variations of model (1). Model (3) scales the basic height-diameter curves by the stand's site index, while model (4) also allows a site index related shift in the form of the height-diameter curve. Model (5) is a form of model (2) in which the basic height-diameter curve is scaled by site index. In applying these models to diameter classes, the dependent variable was defined as the average height of the Dth diameter class for a stand with site index of S.

To fit models (3), (4), and (5) to the uneven-aged height data, they were first transformed by use of natural logarithms to the following:

$$ln(H_D-4.5) = f_1 + f_2 \cdot lnS + f_3 \cdot (D+K)^{-m} \tag{6}$$

$$ln(H_D-4.5) = g_1 + g_2 lnS + g_3 \cdot S \cdot (D+K)^{-m} \tag{7}$$

$$ln(H_D-4.5) = h_1 + h_2 lnS + h_3 lnD + h_4 (lnS)^2 \tag{8}$$

For models (6) and (7), screening runs were then made with values of K ranging from 0.0 to 40.0 and values of m ranging from 0.1 to 4.0. Model (8) was fitted using a single least squares regression run. The three models were then tabulated over D and S to check for reasonableness of behavior, their residuals were plotted over predicted average height to determine homogeneity of variance, and the residuals were also examined for normality. The final model form selected was (6) with K = 35.0 and n = 2.0. The residuals about this model were normally distributed with homogeneous variance, and therefore the lognormal bias correction proposed by Oldham (1965) and Baskerville (1972) could be used. This correction consisted of adding one-half the mean-squared error (0.030203189) to the intercept of model (6) and then taking the antilog of model (6) to obtain the final uneven-aged height model. As an index of fit, the RMSQR for the final log model was 0.1270.

APPENDIX I

EXAMINATION OF STATISTICAL TESTS USED IN VALIDATION

To pursue possible statistical procedures for testing differences between actual and predicted diameter distributions, I conducted a literature review of the validation techniques used in operations research. Of those methods described by Mihram (1972a, 1972b), Van Horn (1971), and Kleijnen (1974), two seemed appropriate for additional evaluation--the chi-square "goodness-of-fit" test and ordinary least squares regression.

The chi-square "goodness-of-fit" test statistic can be computed as:

$$\chi^2 = \sum_{i=1}^{K} \frac{(N_i - np_i)^2}{np_i}$$

where

χ^2 = computed test statistic

n = total number of observations

N_i = number of observations that fell into the ith class

p_i = theoretical probablity of an observation falling into the ith class

k = total number of classes.

This statistic is normally used to test the frequency distribution of test data against a theoretical distribution (Snedecor and Cochran 1967; Kmenta 1971). Therefore, it seemed to be a natural statistic for testing an observed diameter distribution against a theoretical (or predicted) diameter distribution. An examination of the assumptions underlying this test, however, revealed problems that render the usage of the chi-square "goodness-of-fit" statistic for testing diameter distributions questionable, if not inappropriate.

The foundation of the chi-square test is the multinomial distribution. If the assumptions of multinomial distribution can be met, then the limiting distribution of the chi-square "goodness-of-fit" statistic is the chi-square distribution. The two assumptions of the multinomial distribution of interest are: (1) the probability of an observation being in the ith class, p_i, is fixed; and (2) the number of observed values in each class must sum to n (Mood and others 1974; Kendall and Stuart 1973).

Translating these assumptions to the problem at hand, the first assumption would state that the probability of a tree being in a specified diameter class is fixed. While this assumption appears to be unreasonable, it may be possible to define the p_i's as being conditional upon site and stand conditions. Therefore, p_i would be the probability of a tree being in the ith diameter class at the end of the growth period given the present diameter distribution and the specified site index and time-since-last-cutting.

The second assumption states that the total number of trees in the observed diameter distribution must equal the total number of trees in the predicted diameter distribution. To achieve this condition, upgrowth, ingrowth, mortality, and conversion from blackjack to yellow pine must be predicted exactly--and this is highly unlikely. Therefore, while the problem with the first assumption may be avoidable, violation of the second assumption seems unavoidable. As a result, the use of the chi-square "goodness-of-fit" statistic for testing differences between predicted and actual diameter distributions is judged to be inappropriate.

The basic model for the ordinary least squares regression approach is:

$$Y_i = a + b\hat{Y}_i + \varepsilon_i$$

where

Y_i = actual value of a variable (or actual output of the model)

$\hat{Y}_i$ = predicted value of a variable (or predicted output of the model)

a,b = regression coefficients

ε_i = a random deviate.

An intercept (a) of zero and a slope (b) of one would indicate, presumably, that the predicted output of the model is an unbiased estimate of the actual output of the model (Cohen and Cyert 1961; Kleijnen 1974). If the random deviates are distributed normally with mean zero and hemogeneous variance, then standard tests can be used to determine if the intercept is significantly different from zero, and if the slope is significantly different from one (Draper and Smith 1966). Aigner (1972) has shown that the predictive model $\hat{Y}$ must be deterministic for this approach to be legitimate; and the simulator developed in this study is deterministic.

In applying this technique to actual and predicted number of trees in a diameter class, the *t*-tests for determining the significance of the intercept and slope from zero and one, respectively, did not prove to be very useful for making comparisons and decisions. Tests were found in which the intercept and slope were greatly different from their expected values, but, because of the high degree of impreciseness between actual and predicted values (that is, because of the high MSE of the regression), the tests were "insignificant." Conversely, an intercept and/or slope not greatly different from its expected value was often found to be "significantly" different if a low MSE existed (that is, where predictions were highly precise). This limitation of the regression *t*-tests is the same as attributed by Freese (1960) to the paired *t*-test.

Another statistical test considered for use in validation was the Kolmogorov-Smirnov (K-S) test. The standard K-S test was developed for continuous distributions. Because the diameter distribution data are discrete, application of the standard K-S test to it would have resulted in conservative tests (Conover 1971). A discrete K-S test had been reported in the literature (Conover 1972; Horn 1977), but the time and expense of developing and applying the required computer program was judged too great for this study.

APPENDIX J

DESCRIPTION OF CONTROL CARDS

Field function	Value	Column	Name	Format
Card 1				
Title of job		1-80	TITLE	20A4
Card 2				
Minor's (1964) site index	XXX.X	1-5	S	F5.1
Number of 5-year growth periods in run ($\leq$80)	XX	7-8	NGP	I2
Number of 5-year growth periods since last cutting in stands ($1 \leq TCUT \leq 60$). Set to 60 for virgin stands.	XX	10-11	TCUT	I2
Breast height stand age. Set to zero for uneven-aged stands.	XXX	13-15	AGE	F3.0
Cutting scheme				
No cutting	0	17	JCUT	I1
Removals are standard for all cuts and are designated by diameter classes.	1			
Removals are specified for each cut and are designated by diameter classes.	2			
Removals are specified for each cut and are designated by thinning type.	3			
Cutting specifications				
Residual trees per diameter class is specified in number of trees (to be used with JCUT=1 or 2).	1	19	SCUT	I1
Residual trees per diameter class is specified as a proportion (to be used with JCUT=1 or 2).	2			
Residual total stand basal area is specified (to be used with JCUT=3).	3			
Residual total number of trees in stand is specified (to be used with JCUT=3).	4			
Proportion of mortality to be recovered in thinnings	X.XXXX	21-26	MORREC	F6.4
Proportion of removals to come from below when thinning from both above and below (to be used when JCUT=3 and thinning type =4). Set to 0.0 if JCUT≠3 or thinning type ≠4.	X.XXXX	28-33	PROREM	F6.4

Field function	Value	Column	Name	Format
Card 3				
A "C" punched in columns whose ordinals are the growth periods for which cutting is to occur. Blank if no cutting is to occur.	C or blank	1-80	NC	80A1
Card(s) 4,...,N1				
From 1 to as many as 6 cards				
Initial blackjack pine array				
Note: Omit if no blackjack pine.				
For each card:				
Card type	INITIAL BJ	1-10	CHECK	A4, 6X
Diameter class (≤45)	XX	12-13	ID(1)	I2
Number of trees in diameter class	XXX.XX	15-20	T(1)	F6.2
Diameter class (≤45)	XX	12-23	ID(2)	I2
Number of trees in diameter class	XXX.XX	25-30	T(2)	F6.2
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
Diameter class (≤45)	XX	72-73	ID(7)	I2
Number of trees in diameter class	XXX.XX	75-80	T(7)	F6.2
Card N1+1				
Card type (This card must be present, even if no blackjack pine)	DONE	1-4	CHECK	A4
Card(s) N1+2,...,N2				
From 1 to as many as 6 cards				
Initial yellow pine array				
Note: Omit if no yellow pine				
For each card:				
Card type (Rest of card same as for initial blackjack pine cards)	INITIAL YP	1-10	CHECK	A4, 6X
Card N2+1				
Card type (This card must be present, even if no yellow pine)	DONE	1-4	CHECK	A4
Card(s) N2+2,...,N3				
For JCUT=1 or 2, only (for JCUT=3, go to next definition of these cards)				
From 1 to as many as 6 cards				
Blackjack pine cutting array				
Note: Omit if no blackjack pine cutting				

Field function	Value	Column	Name	Format
For each card:				
Card type	CUTTING BJ	1-10	CHECK	A4, 6X
Diameter class (≤45)	XX	12-13	ID(1)	I2
If SCUT=1, number of residual trees in diameter class	XXX.XX	15-20	T(1)	F6.2
If SCUT=2, proportion of diameter class to be uncut	X.XXXX	15-20	T(1)	F6.4
Diameter class (≤45)	XX	22-23	ID(2)	I2
If SCUT=1, number of residual trees in diameter class	XXX.XX	25-30	T(2)	F6.2
If SCUT=2, proportion of diameter class to be uncut	X.XXXX	25-30	T(2)	F6.4
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
Diameter class (≤45)	XX	72-73	ID(7)	I2
If SCUT=1, number of residual trees in diameter class	XXX.XX	75-80	T(7)	F6.2
If SCUT=2, proportion of diameter class to be uncut	X.XXXX	75-80	T(7)	F6.4
Repeat the above card type until all diameter classes are defined.				

Card(s) N2+2,...,N3

For JCUT=3, only (for JCUT 1 or 2, go to previous definition of these cards)

From 1 to as many as 27 cards

Field function	Value	Column	Name	Format
Thinning array				
Note: Omit if no thinning				
For each card:				
Card type	THINNINGS	1-9	CHECK	A4, 5X
Growth period for thinning (Order of thinnings on card need not be sequential)	XX	12-13	ID(1)	I2
Thinning type:				
Thinning from below[7]	1	15	ID(2)	I1
Thinning from above[8]	2			
Thinning from below and above	3			
Thinning proportionally across all diameter classes[9]	4			

[7]Thinning from below is the removal of trees from the smallest diameter class first.

[8]Thinning from above is the removal of trees from the largest diameter class first.

[9]Proportional or random thinning is the removal of basal area (SUC=3) or trees (SCUT=4) proportionally from all diameter classes. The form of the basal area or diameter distribution after thinning will be the same as that before thinning.

Field function	Value	Column	Name	Format
If SCUT=3, residual blackjack pine Basal area after thinning	XXX.XX	18-23	T(1)	F6.2
If SCUT=4, residual blackjack pine Number of trees after thinning	XXXX.X	18-23	T(1)	F6.1
If SCUT=3, residual yellow pine Basal area after thinning	XXX.XX	25-30	T(2)	F6.2
If SCUT=4, residual yellow pine Number of trees after thinning	XXXX.X	25-30	T(2)	F6.1
Growth period for thinning	XX	32-33	ID(3)	I2
Thinning type	X	35	ID(4)	I1
If SCUT=3, residual blackjack pine Basal area after thinning	XXX.XX	38-43	T(3)	F6.2
If SCUT=4, residual blackjack pine Number of trees after thinning	XXXX.X	38-43	T(3)	F6.1
If SCUT=3, residual yellow pine Basal area after thinning	XXXX.X	45-50	T(4)	F6.2
If SCUT=4, residual yellow pine Number of trees after thinning	XXXX.X	45-50	T(4)	F6.1
Growth period for thinning	XX	52-53	ID(5)	I2
Thinning type	X	55	ID(6)	I1
If SCUT=3, residual blackjack pine Basal area after thinning	XXX.XX	58-63	T(5)	F6.2
If SCUT=4, residual blackjack pine Number of trees after thinning	XXXX.X	58-63	T(5)	F6.1
If SCUT=3, residual yellow pine Basal area after thinning	XXX.XX	65-70	T(6)	F6.2
If SCUT=4, residual yellow pine Number of trees after thinning	XXXX.X	65-70	T(6)	F6.1

Repeat the above card type until all thinnings specified on card 3 are defined.

Field function	Value	Column	Name	Format
Card N3+1				
Card type (This card must be present, even if no blackjack cutting or thinning)	DONE	1-4	CHECK	A4
Card(s) N3+2,...,N4 For JCUT=1 or 2 only From 1 to as many as 6 cards				
Yellow pine cutting array Note: Omit if JCUT=3 or if no yellow pine cutting.				
For each card:				
Card type (Rest of card same as for blackjack pine cutting cards)	CUTTING YP	1-10	CHECK	A4, 6X
Repeat the above card type until all diameter classes are defined.				
Card N4+1				
Card type (This card must be present *if* JCUT=1 or 2)	DONE	1-4	CHECK	A4
If JCUT=2, then cards (N2+2) to (N4+1) are repeated for as many times as there are "C's" punched in card 3.				
Card N4+2				
Card type if another growth analysis is to be done	CONTINUE	1-8	CHECK	A4
Card type if last growth analysis	FINISH	1-6	CHECK	A4

If CONTINUE, then repeat cards 1 to (N4+1) for the new analysis.

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This paper describes the construction and validation of a simulator for predicting even-aged and uneven-aged stand development for the ponderosa pine/Arizona fescue habitat type of the Southwest. The resulting simulator characterizes the stand by the number of trees in 1-inch diameter classes for two vigor components of the stand. Stand dynamics are represented by models for predicting upgrowth, mortality, vigor class conversion, and ingrowth.

KEYWORDS: *Pinus ponderosa*, growth, mortality, ingrowth, conversion, forest management, linear regression, nonlinear regression, validation.

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